Parton contents of a lepton at high energies

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In collaboration with Tao Han and Keping Xie

[T. Han, Y. Ma, K.Xie 2007.14300]
[T. Han, Y. Ma, K.Xie 2103.09844]
A little bit background (I): What is “parton”?

Recall the hadron colliders: \( pp \) collision at the Tevatron or the LHC

**Factorization formalism:** PDFs \( \otimes \) partonic cross sections

\[
\sigma(AB \rightarrow X) = \sum_{a,b} \int dx_a dx_b f_{a/A}(x_a, Q)f_{b/B}(x_b, Q) \hat{\sigma}(ab \rightarrow X)
\]

- \( a, b \) are the “partons” from the beam particles \( A \) and \( B \).
- \( f_{a/A}, f_{b/B} \) are the probabilities to find a parton \( a \) (\( b \)) from the beam particle \( A \) (\( B \)) with a momentum fraction \( x_a \) (\( x_b \)).
A little bit background (II): PDFs of a lepton

“Equivalent photon approximation (EPA)”

Treat photon as a parton constituent in the electron

$$\sigma(\ell^- + a \rightarrow \ell^- + X) = \int dx f_{\gamma/\ell} \hat{\sigma}(\gamma a \rightarrow X)$$

$$f_{\gamma/\ell, \text{EPA}}(x_\gamma, Q^2) = \frac{\alpha}{2\pi} \frac{1 + (1 - x_\gamma)^2}{x_\gamma} \ln \frac{Q^2}{m_\ell^2}$$

Applications at lepton colliders

- Production cross sections

$$\sigma(\ell^+ \ell^- \rightarrow F + X) = \int_{\tau_0}^1 d\tau \sum_{ij} \frac{dL_{ij}}{d\tau} \hat{\sigma}(ij \rightarrow F), \ \tau = \hat{s} / s$$

- Partonic luminosities

$$\frac{dL_{ij}}{d\tau} = \frac{1}{1 + \delta_{ij}} \int_\tau^1 \frac{d\xi}{\xi} \left[ f_i(\xi, Q^2) f_j \left( \frac{\tau}{\xi}, Q^2 \right) + (i \leftrightarrow j) \right]$$

Extra terms:

[Frixione, Mangano, Nason, Ridolfi 2103.09844]
[Budnev, Ginzburg, Meledin, Serbo, Phys. Rept.(1975)]
An anatomical study: Why EWPDF?

Fixed order (FO) VS EPA for $e^- \mu^+ \rightarrow e^- W^+ \bar{\nu}_\mu$

- Feynman diagrams

- Q: Is it safe to consider only photon?

- Things to know:
  - Gauge invariance requires all diagrams
  - Unphysical results appear at high energies

- Need more PDFs, we should go beyond EPA!

Q: Why EPA works?
A: Forward $\gamma$ dominates

Q: Why EPA works?
A: Forward $\gamma$ dominates
A possible high-energy lepton (muon) collider

Why muon colliders?

- **Leptons** are the ideal probes of short-distance physics
  -Nearly all the energy is stored in the colliding partons
  -High-energy physics probed with much smaller collider energy
- **Electrons** radiate too much
- A $s$-channel Higgs factory: Direct measurements on $y_\mu$ and $\Gamma_H$
- **Multi-TeV muon colliders**:
  -Center of mass energy $3 - 15$ TeV and the more speculative $E_{cm} = 30$ TeV
  -Luminosity: $\mathcal{L} = (E_{cm}/10\text{ TeV})^2 \times 10\text{ab}^{-1}$
  -New particle mass coverage $M \sim (0.5 - 1)E_{cm}$
  -Great accuracies for $WWH$, $WWHH$, $H^3$, $H^4$
  -\ldots

Muon Collider Physics Potential Pillars

- **Direct search of heavy particles**
  -SUSY-inspired, WIMP, VBF production, 2->1

- **High rate indirect probes**
  -Higgs single and self-couplings, rare Higgs decays, exotic decays

- **High energy probes**
  -difermion, diboson, EFT, Higgs compositeness
A high-energy lepton collider at first glance (I)

What do people expect from a high-energy lepton collider?

Some “commonsense”:

- The annihilations decrease as $1/s$.
- ISR needs to be considered, which can give over 10% enhancement.
- The fusions increase as $\ln^p(s)$, which take over at high energies.
- The large collinear logarithm $\ln \left( s/m_\ell^2 \right)$ needs to be resummed, set $Q = \sqrt{s}/2$,
- $\gamma\gamma \to W^+W^-$ production has the largest cross section.

[T. Han, Y. Ma, K.Xie 2007.14300]
What are the dominant processes at a high-energy lepton collider?

- Leading-order: $\ell^+\ell^- \rightarrow \ell^+\ell^-, \tau^+\tau^-, q\bar{q}, W^+W^-$, and $\gamma\ell \rightarrow \gamma\ell$
- $\gamma\gamma$ scatterings: $\gamma\gamma \rightarrow \tau^+\tau^-, q\bar{q}, W^+W^-$

Need some cuts:

- Detector angle: $\theta_{\text{cut}} = 5^\circ (10^\circ) \iff |\eta| < 3.13(2.44)$
- Threshold: $m_{ij} > 20$ GeV
- Need a $p_T$ cut to separate from the nonperturbative hadronic production

\[ p_T > (4 + \sqrt{s}/3 \text{ TeV}) \text{ GeV} \]
Go beyond the EPA at high-energy lepton colliders

We have been doing:

- $\ell^+\ell^-$ annihilation

- EPA and ISR

- "Effective W Approx." (EWA)

We will add:

- Above $\mu_{QCD}$: QED$\otimes$QCD
  
  q/g emerge

- Above $\mu_{EW} = M_Z$: EW$\otimes$QCD
  
  EW partons emerge

In the end, everything is parton, i.e. the full SM PDFs.
The PDF evolution: DGLAP

- The DGLAP equations

\[
\frac{df_i}{d \log Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P_{ij}^I \otimes f_j
\]

- The initial conditions

\[
f_{\ell/\ell}(x, m_\ell^2) = \delta(1 - x)
\]

- Three regions and two matchings
  - \( m_\ell < Q < \mu_{\text{QCD}} \): QED
  - \( Q = \mu_{\text{QCD}} \lesssim 1 \text{ GeV} \): \( f_q \propto P_{q\gamma} \otimes f_\gamma, f_g = 0 \)
  - \( \mu_{\text{QCD}} < Q < \mu_{\text{EW}} \): QED\( \otimes \)QCD
  - \( Q = \mu_{\text{EW}} = M_Z \): \( f_\nu = f_t = f_W = f_Z = f_{\gamma Z} = 0 \)
  - \( \mu_{\text{EW}} < Q \): EW\( \otimes \)QCD.

\[
\begin{pmatrix}
    f_B \\
    f_W^3 \\
    f_{BW^3}
\end{pmatrix}
= \begin{pmatrix}
    c_W^2 & s_W^2 & -2c_W s_W \\
    s_W^2 & c_W^2 & 2c_W s_W \\
    c_W s_W & -c_W s_W & c_W^2 - s_W^2
\end{pmatrix}
\begin{pmatrix}
    f_\gamma \\
    f_Z \\
    f_{\gamma Z}
\end{pmatrix}
\]

- We work in the \((B, W)\) basis. The technical details can be referred to the backup slides.
Solving the DGLAP: Singlet and Non-singlet PDFs

The singlets

\[ f_L = \sum_{i=e,\mu,\tau} (f_{\ell_i} + f_{\bar{\ell}_i}), \quad f_U = \sum_{i=u,c} (f_{u_i} + f_{\bar{u}_i}), \quad f_D = \sum_{i=d,s,b} (f_{d_i} + f_{\bar{d}_i}) \]

The non-singlets

- The only non-trivial singlet \( f_{e,NS} = f_e - f_{\bar{e}} \)
- the leptons
  \[ f_{\ell_i,NS} = f_{\ell_i} - f_{\bar{\ell}_i} (i = 2,3), \quad f_{\ell,12} = f_e - f_\mu, \quad f_{\ell,13} = f_e - f_\tau; \]
- the up-type quarks
  \[ f_{u_i,NS} = f_{u_i} - f_{\bar{u}_i}, \quad f_{u,12} = f_u - f_c; \]
- and the down-type quarks
  \[ f_{d_i,NS} = f_{d_i} - f_{\bar{d}_i}, \quad f_{d,12} = f_d - f_s, \quad f_{d,13} = f_d - f_b. \]

Reconstruction:

\[ f_e = \frac{f_L + (2N_\ell - 1)f_{e,NS}}{2N_\ell}, \quad f_\bar{e} = f_\mu = f_{\bar{\mu}} = f_\tau = f_{\bar{\tau}} = \frac{f_L - f_{e,NS}}{2N_\ell}. \]

\[ f_u = f_{\bar{u}} = f_c = f_{\bar{c}} = \frac{f_U}{2N_u}, \quad f_d = f_{\bar{d}} = f_s = f_{\bar{s}} = f_b = f_{\bar{b}} = \frac{f_D}{2N_d}. \]
The QED⊗QCD case

- The singlets and gauge bosons

\[
\frac{d}{d\log Q^2} \begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix} = \begin{pmatrix} P_{\ell\ell} & 0 & 0 & 2N_{\ell}P_{\ell\gamma} & 0 \\ 0 & P_{uu} & 0 & 2N_uP_{u\gamma} & 2N_uP_{ug} \\ 0 & 0 & P_{dd} & 2N_dP_{d\gamma} & 2N_dP_{dg} \\ P_{\gamma\ell} & P_{\gamma u} & P_{\gamma d} & P_{\gamma\gamma} & 0 \\ 0 & P_{gu} & P_{gd} & 0 & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix}
\]

- The non-singlets

\[
\frac{d}{d\log Q^2} f_{NS} = P_{ff} \otimes f_{NS}.
\]

- The averaged momentum fractions of the PDFs: \( f_{\ell\text{val}}, f_\gamma, f_{\ell\text{sea}}, f_q, f_g \)

\[
\langle x_i \rangle = \int x f_i(x) dx, \quad \sum_i \langle x_i \rangle = 1
\]

\[
\frac{\langle x_q \rangle}{\langle x_{\ell\text{sea}} \rangle} \lesssim \frac{N_c \left[ \sum_i (e_{ui}^2 + e_{\bar{u}i}^2) + \sum_i (e_{di}^2 + e_{\bar{d}i}^2) \right]}{e_{\ell\text{val}}^2 + \sum_{i \neq \ell\text{val}} (e_{\ell i}^2 + e_{\bar{\ell} i}^2)} = \frac{22/3}{5}
\]
The QED⊗QCD PDFs for lepton colliders

- **Electron PDFs:** $f_{e_{\text{val}}}, f_\gamma, f_{\ell_{\text{sea}}, f_q, f_g}$
- **Scale uncertainty:** 10% for $f_{g/e}$
- **The averaged momentum fractions**
  
  $\langle x_i \rangle = \int x f_i(x) \, dx$

<table>
<thead>
<tr>
<th>$Q(e^\pm)$</th>
<th>$e_{\text{val}}$</th>
<th>$\gamma$</th>
<th>$\ell_{\text{sea}}$</th>
<th>$q$</th>
<th>$g$</th>
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</thead>
<tbody>
<tr>
<td>30 GeV</td>
<td>96.6</td>
<td>3.20</td>
<td>0.069</td>
<td>0.080</td>
<td>0.023</td>
</tr>
<tr>
<td>50 GeV</td>
<td>96.5</td>
<td>3.34</td>
<td>0.077</td>
<td>0.087</td>
<td>0.026</td>
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<tr>
<td>$M_Z$</td>
<td>96.3</td>
<td>3.51</td>
<td>0.085</td>
<td>0.097</td>
<td>0.028</td>
</tr>
</tbody>
</table>

- **Muon PDFs:** $f_{\mu_{\text{val}}}, f_\gamma, f_{\ell_{\text{sea}}, f_q, f_g}$
- **Scale uncertainty:** 20% for $f_{g/\mu}$
- **The averaged momentum fractions**
  
  $\langle x_i \rangle = \int x f_i(x) \, dx$

<table>
<thead>
<tr>
<th>$Q(\mu^\pm)$</th>
<th>$\mu_{\text{val}}$</th>
<th>$\gamma$</th>
<th>$\ell_{\text{sea}}$</th>
<th>$q$</th>
<th>$g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 GeV</td>
<td>98.2</td>
<td>1.72</td>
<td>0.019</td>
<td>0.024</td>
<td>0.0043</td>
</tr>
<tr>
<td>50 GeV</td>
<td>98.0</td>
<td>1.87</td>
<td>0.023</td>
<td>0.029</td>
<td>0.0051</td>
</tr>
<tr>
<td>$M_Z$</td>
<td>97.9</td>
<td>2.06</td>
<td>0.028</td>
<td>0.035</td>
<td>0.0062</td>
</tr>
</tbody>
</table>
The DGLAP for the full SM

\[
\frac{d}{dL} \left( \begin{array}{c}
  f_L^{0\pm} \\
  f_Q^{0\pm} \\
  f_E^{0\pm} \\
  f_U^{0\pm} \\
  f_D^{0\pm} \\
  f_B^{0\pm} \\
  f_W^{0\pm} \\
  f_g^{0\pm}
\end{array} \right) = 
\left( \begin{array}{cccccc}
  P_{LL}^{0\pm} & 0 & 0 & 0 & 0 & P_{LB}^{0\pm} \\
  0 & P_{QQ}^{0\pm} & 0 & 0 & 0 & P_{QB}^{0\pm} \\
  0 & 0 & P_{EE}^{0\pm} & 0 & 0 & P_{EB}^{0\pm} \\
  0 & 0 & 0 & P_{UU}^{0\pm} & 0 & P_{UB}^{0\pm} \\
  P_{BL}^{0\pm} & P_{BQ}^{0\pm} & P_{BE}^{0\pm} & P_{BU}^{0\pm} & P_{BB}^{0\pm} & 0 \\
  P_{WL}^{0\pm} & P_{WQ}^{0\pm} & 0 & 0 & 0 & P_{WW}^{0\pm} \\
  0 & P_{gQ}^{0\pm} & 0 & P_{gU}^{0\pm} & P_{gD}^{0\pm} & 0 \\
  0 & 0 & 0 & 0 & 0 & P_{gg}^{0\pm}
\end{array} \right) \otimes 
\left( \begin{array}{c}
  f_L^{0\pm} \\
  f_Q^{0\pm} \\
  f_E^{0\pm} \\
  f_U^{0\pm} \\
  f_D^{0\pm} \\
  f_B^{0\pm} \\
  f_W^{0\pm} \\
  f_g^{0\pm}
\end{array} \right)
\]

\[
\frac{d}{dL} \left( \begin{array}{c}
  f_L^{1\pm} \\
  f_Q^{1\pm} \\
  f_E^{1\pm} \\
  f_U^{1\pm} \\
  f_D^{1\pm} \\
  f_B^{1\pm} \\
  f_W^{1\pm} \\
  f_{BW}^{1\pm}
\end{array} \right) = 
\left( \begin{array}{cccccc}
  P_{LL}^{1\pm} & 0 & 0 & P_{LM}^{1\pm} \\
  0 & P_{QQ}^{1\pm} & P_{QM}^{1\pm} & 0 \\
  P_{WL}^{1\pm} & P_{WQ}^{1\pm} & P_{WW}^{1\pm} & 0 \\
  P_{ML}^{1\pm} & P_{MQ}^{1\pm} & 0 & P_{MM}^{1\pm}
\end{array} \right) \otimes 
\left( \begin{array}{c}
  f_L^{1\pm} \\
  f_Q^{1\pm} \\
  f_E^{1\pm} \\
  f_U^{1\pm} \\
  f_D^{1\pm} \\
  f_B^{1\pm} \\
  f_W^{1\pm} \\
  f_{BW}^{1\pm}
\end{array} \right)
\]

\[
\frac{d}{dL} f_{W}^{2\pm} = P_{WW}^{2\pm} \otimes f_{WW}^{2\pm}
\]

The splitting functions can be constructed in terms of Refs.

[Chen et al. 1611.00788, Bauer et al. 1703.08562,1808.08831]
EWPDFs of a lepton

- The sea leptonic and quark PDFs

\[ \nu = \sum_i (\nu_i + \bar{\nu}_i), \ \ell_{\text{sea}} = \bar{\mu} + \sum_{i \neq \mu} (\bar{\ell}_i + \ell_i), \ q = \sum_{i=d}^t (q_i + \bar{q}_i) \]

There is even neutrino due to the EW sector, **everything is a parton!**

- All SM particles are partons [T. Han, Y. Ma, K.Xie 2007.14300]
- \( \bar{W}_L \) does not evolve: **Bjorken-scaling restoration.**
- The EW correction is not small: \( \sim 50\% \ (100\%) \) for \( f_d/e \ (f_d/\mu) \) due to the relatively large SU(2) gauge coupling. [T. Han, Y. Ma, K.Xie 2103.09844]
- Scale uncertainty: \( \sim 15\% \ (20\%) \) between \( Q = 3 \) TeV and \( Q = 5 \) TeV
Parton luminosities at high-energy lepton colliders

Consider a 3 TeV $e^+e^-$ machine and a 10 TeV $\mu^+\mu^-$ machine

- Partonic luminosities for $\ell^+\ell^-$, $\gamma\ell$, $\gamma\gamma$, $qq$, $\gamma q$, $\gamma g$, $gq$, and $gg$

The partonic luminosity of $\gamma g + \gamma q$ is $\sim 50\%$ ($20\%$) of the $\gamma\gamma$ one

The partonic luminosities of $qq$, $gq$, and $gg$ are $\sim 2\%$ ($0.5\%$) of the $\gamma\gamma$ one

Given the stronger QCD coupling, sizable QCD cross sections are expected.

- Scale uncertainty is $\sim 20\%$ ($50\%$) for photon (gluon) initiated processes.
Jet production of possible lepton colliders (I)

- Large photon induced non-perturbative hadronic production
  
  [Drees and Godbole, PRL 67 1189, hep-ph/9203219]
  

  - $\sigma_{\gamma\gamma}$ may reach micro-barns level at TeV c.m. energies
  - $\sigma_{\ell\ell}$ may reach nano-barns, after folding in the $\gamma\gamma$ luminosity

- The events populate at low $p_T$ regime

  So we can separate from this non-perturbative range via a $p_T$ cut.

[Image of plots showing $p_T$ distributions for Pythia and SLAC samples]

[T. Barklow, D. Dannheim, M. O. Sahin, and D. Schulte, LCD-2011-020]
Jet production at possible lepton colliders (II)

- High-$p_T$ range $[p_T > (4 + \sqrt{s}/3 \text{ TeV}) \text{ GeV}]$: perturbatively computable
  
  \begin{align*}
  \gamma \gamma & \rightarrow q \bar{q}, \ \gamma g \rightarrow q \bar{q}, \ \gamma q \rightarrow gq, \\
  qq & \rightarrow qq \ (gg), \ gq \rightarrow gq \ \text{and} \ gg \rightarrow gg \ (q \bar{q}).
\end{align*}

- Large $\alpha_s \ln(Q^2)$ brings a $6\% \sim 15\% \ (30\% \sim 40\%)$ enhancement if $Q = 2Q$.

Including the QCD contribution leads to much larger total cross section.

- $gg$ initiated cross sections are large for its large multiplicity;
- $gq$ initiated cross sections are large for its large luminosity.
- $\gamma \gamma$ initiated cross sections here are smaller than the EPA results.
Refresh the picture of high-energy lepton colliders

What is the dominant process at a high-energy muon collider?

- Quark/gluon initiated jet production dominates

Before:

After:
Di-jet distributions at a muon collider

Rather a conservative set up: $\theta = 10^\circ$

- Some physics:
  - Two different mechanisms: $\mu^+\mu^-$ annihilation VS Fusion processes
    - Annihilation is more than 2 orders of magnitude smaller than fusion process.
    - Annihilation peaks at $m_{ij} \sim \sqrt{s}$;
    - Fusion processes peak near $m_{ij}$ threshold.
    - Annihilation is very central, spread out due to ISR;
    - Fusion processes spread out, especially for $\gamma q$ and $\gamma g$ initiated ones.

![Graphs showing di-jet distributions for different processes](image-url)
Jet production dominates over $WW$ production until $p_T > 60$ GeV;

$WW$ production takes over around energy $\sim 200$ GeV.

QCD contributions are mostly forward-backward; $\gamma\gamma$, $\gamma q$, and $\gamma g$ initiated processes are more isotropic.
The EW parton luminosities of a 30 TeV muon collider

- **Production cross sections**

\[
\sigma(\ell^+ \ell^- \to F + X) = \int_{\tau_0}^{1} d\tau \sum_{ij} \frac{d\mathcal{L}_{ij}}{d\tau} \hat{\sigma}(ij \to F), \quad \tau = \frac{s}{\hat{s}}
\]

- **Partonic luminosities**

\[
\frac{d\mathcal{L}_{ij}}{d\tau} = \frac{1}{1 + \delta_{ij}} \int_{\tau}^{1} \frac{d\xi}{\xi} \left[ f_i(\xi, Q^2)f_j \left( \frac{\tau}{\xi}, Q^2 \right) + (i \leftrightarrow j) \right]
\]

![Graph showing the production cross sections and partonic luminosities for various processes at \(\sqrt{s} = 30\) TeV.]
One example: \( t\bar{t} \) production at a muon collider
Other processes: $W^+ W^-, ZH, HH, t\bar{t}H$
The full picture: Semi-inclusive processes

Just like in hadronic collisions:

\[ \mu^+ \mu^- \rightarrow \text{exclusive particles} + \text{remnants} \]
Summary and prospects

**EWPDF:**
- At very high energies, the collinear splittings dominate. **All SM particles should be treated as partons that described by proper PDFs.**
  - The large collinear logarithm needs to be resummed via solving the DGLAP equations, so the QCD partons (quarks and gluons) emerge.
  - When $Q > M_Z$, the EW splittings are activated: the EW partons appear, and the existing QED⊗QCD PDFs may receive big corrections.

**For a high-energy lepton (muon) collider:**
- There are many things to work on: SUSY, DM, Higgs, etc.
- The parton fusion processes exceed the $\ell^+\ell^-$ annihilation, where we should employ the EWPDFs [T. Han, Y. Ma, K.Xie 2007.14300]
- **The main background of is the jet production:**
  - Low $p_T$ range: non-perturbative $\gamma\gamma$ initiated hadronic production dominates
    [Chen, Barklow, and Peskin, hep-ph/9305247; Drees and Godbole, PRL 67, 1189,T. Barklow, etal, LCD-2011-020]
  - High $p_T$ range, $q$ and $g$ initiated jet production dominates
    [T. Han, Y. Ma, K.Xie 2103.09844]
- EWPDF allows to determine the contributions from different partons and their different polarizations
- One should look at the semi-inclusive processes
The novel features of the EW PDFs

- The EW PDFs must be polarized due to the chiral nature of the EW theory
  \[ f_{V+/A+} \neq f_{V-/A-}, \quad f_{V+/A-} \neq f_{V-/A+}, \]
  \[ \hat{\sigma}(V_+ B_+) \neq \hat{\sigma}(V_- B_-), \quad \hat{\sigma}(V_+ B_-) \neq \hat{\sigma}(V_- B_+) \]

  We are not able to factorize the cross sections in an unpolarized form.

  \[ \sigma \neq f_{V/A} \hat{\sigma}(VB), \quad f_{V/A} = \frac{1}{2} \sum_{\lambda, s_1} f_{V\lambda/A s_1}, \quad \hat{\sigma}(VB) = \frac{1}{4} \sum_{\lambda, s_2} \hat{\sigma}(V\lambda B s_2) \]

- The interference gives the mixed PDFs

  [Bauer '17, '18, Manohar '18, TH '16.]

  \[ f_{\gamma Z} \sim \langle \Omega | A^{\mu\nu} Z_{\mu\nu} | \Omega \rangle + \text{h.c.}, \]

  similarly for \( f_{hZ_L} \).

- Bloch-Nordsieck theorem violation due to the non-cancelled divergence in \( f \rightarrow f' V \): cutoff \( M_V / Q \) or redefinition

  \[ f_1 \sim f_e + f_\nu, \quad f_3 \sim f_e - f_\nu \]
Return to the beginning: $e^- \mu^+ \rightarrow e^- W^+ \bar{\nu}_\mu$

- EWA: $f_{V\lambda/e_s^\pm}(x, Q) = \frac{1}{8\pi^2} g_1 g_2 P_{V\lambda/e_s^\pm}(x) \log \left( \frac{Q^2}{m_Z^2} \right)$

$$g_L = \frac{g_2}{c_W} \left( -\frac{1}{2} + s_W^2 \right) < 0, \quad g_R = \frac{g_2}{c_W} s_W^2 > 0, \quad g_e = -e$$

<table>
<thead>
<tr>
<th></th>
<th>$e_L^-$</th>
<th>$e_R^-$</th>
<th>$e_L^+$</th>
<th>$e_R^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_-$</td>
<td>$g_L^2 \frac{1}{x}$</td>
<td>$g_R^2 \frac{(1-x)^2}{x}$</td>
<td>$g_L^2 \frac{(1-x)^2}{x}$</td>
<td>$g_R^2 \frac{1}{x}$</td>
</tr>
<tr>
<td>$Z_+$</td>
<td>$g_L^2 \frac{(1-x)^2}{x}$</td>
<td>$g_R^2 \frac{1}{x}$</td>
<td>$g_L^2 \frac{1}{x}$</td>
<td>$g_R^2 \frac{(1-x)^2}{x}$</td>
</tr>
<tr>
<td>$\gamma Z_-$</td>
<td>$g_e g_L \frac{1}{x}$</td>
<td>$g_e g_R \frac{(1-x)^2}{x}$</td>
<td>$g_e g_L \frac{(1-x)^2}{x}$</td>
<td>$g_e g_R \frac{1}{x}$</td>
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<tr>
<td>$\gamma Z_+$</td>
<td>$g_e g_L \frac{(1-x)^2}{x}$</td>
<td>$g_e g_R \frac{1}{x}$</td>
<td>$g_e g_L \frac{1}{x}$</td>
<td>$g_e g_R \frac{(1-x)^2}{x}$</td>
</tr>
</tbody>
</table>

- The contribution of the mixed PDF $f_{\gamma Z}$ can be either constructive or destructive

$$\sigma = \sum_{\lambda,s_1,s_2} f_{V\lambda/e_s^-} \hat{\sigma}(V\lambda\mu_{s_2}^+ \rightarrow W^+ \bar{\nu}_\mu)$$
EW physics at high energies

- At high energies, every particle become massless

\[ \frac{v}{E} : \frac{v}{100 \text{ TeV}} \sim \frac{\Lambda_{\text{QCD}}}{100 \text{ GeV}}, \frac{v}{E}, \frac{m_t}{E}, \frac{M_W}{E} \to 0! \]

- The splitting phenomena dominate due to large log enhancement

- The EW symmetry is restored: \( SU(2)_L \times U(1)_Y \) unbroken

- Goldstone Boson Equivalence:

\[ \varepsilon_L^\mu(k) = \frac{E}{M_W} (\beta_W, \hat{k}) \simeq \frac{k^\mu}{M_W} + \mathcal{O}(\frac{M_W}{E}) \]

The violation terms is power counted as \( v/E \to \text{QCD higher twist effects} \)

\[ \Lambda_{\text{QCD}} / Q \] [G. Cuomo, A. Wulzer, arXiv:1703.08562; 1911.12366].

- We mainly focus on the splitting phenomena, which can be factorized and resummed as the EW PDFs in the ISR, and the Fragments/Parton Shower in the FRS.

- Other interesting aspects: the polarized EW boson scattering, top-Yukawa coupling effect
Factorization of the EW splittings

\[ \frac{d\sigma}{dzdk_T^2} \approx \frac{1}{16\pi^2} \frac{z\bar{z}|M^{(\text{split})}|^2}{(k_T^2 + \bar{z}m_B^2 + zm_C^2 - z\bar{z}m_A^2)^2}, \quad \bar{z} = 1 - z \]

- The dimensional counting: \(|M^{(\text{split})}|^2 \sim k_T^2 \text{ or } m^2\)
- To validate the factorization formalism
  - The observable \(\sigma\) should be infra-red safe
  - Leading behavior comes from the collinear splitting

[Ciafaloni et al., hep-ph/0004071; 0007096; C. Bauer, Ferland, B. Webber et al., arXiv:1703.08562;1808.08831]

[A. Manohar et al., 1803.06347; T. Han, J. Chen & B. Tweedie, arXiv:1611.00788]
### Splitting functions: EW

- **Starting from the unbroken phase:** all massless
  \[ \mathcal{L}_{SU(2) \times U(1)} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_\phi + \mathcal{L}_f + \mathcal{L}_{\text{Yukawa}} \]

- **Particle contents:**
  - Chiral fermions: \( f_{L,R} \)
  - Gauge bosons: \( B, W^0, \pm \)
  - Higgs: \( H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \sqrt{2} (h - i\phi^0) \end{pmatrix} \)

- **Splitting functions** [See Ciafaloni et al. hep-ph/0505047, Chen et al. 1611.00788 for complete lists.]

<table>
<thead>
<tr>
<th>( f_s = L, R \rightarrow )</th>
<th>( V_T f_s^{(')} )</th>
<th>( [BW]^0_T f_s )</th>
<th>( H^0* f_{-s} )</th>
<th>( \phi^\pm f_{-'s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g^2_V (Q_{f_s}^V)^2 )</td>
<td>( g_1 g_2 \ Y_{f_s} T_{f_s}^3 )</td>
<td>( y^2_{f_R^{(')} )</td>
<td>( y^2_{f_{R}^{(')} )</td>
<td></td>
</tr>
<tr>
<td>Infrared, collinear singularites (( P_{gq} ))</td>
<td>Collinear singularity chirality-flip, Yukawa</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Corrections to the GET in the EWSB

- New fermion splitting: $P \sim \frac{v^2}{k_T^2} \frac{d k_T^2}{k_T^2}$

- $V_L$ is of IR, $h$ has no IR

<table>
<thead>
<tr>
<th>$f_s$</th>
<th>$V_L f_s^{(l)} \ (V \neq \gamma)$</th>
<th>$h f_s$</th>
<th>$V_T f_s^{(l)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{16\pi^2} \frac{v^2}{k_T^4} \frac{1}{z}$</td>
<td>$\frac{1}{16\pi^2} \frac{v^2}{k_T^4}$</td>
<td>$\frac{1}{16\pi^2} \frac{v^2}{k_T^4}$</td>
<td></td>
</tr>
</tbody>
</table>

- Chirality conserving non-zero for massless $f$

- Chirality flipping $\sim m_f$

- The PDFs for $W_L/Z_L$ behaves as constants, which does not run at the leading log: "Bjorken scaling" restoration (higher-twist effects)

$$f_{V_L/f}(x, Q^2) \sim \alpha \frac{1-x}{x}$$
PDFs and Fragmentations (parton showers)

- Initial state radiation (ISR), PDFs (DGLAP):

\[
f_B(z, \mu^2) = \sum_A \int z \frac{d\xi}{\xi} f_A(\xi) \int_{m^2}^{\mu^2} d\mathcal{P}_{A \rightarrow B+C} \left(\frac{z}{\xi}, k_T^2\right)
\]

\[
\frac{\partial f_B(z, \mu^2)}{\partial \mu^2} = \sum_A \int z \frac{d\xi}{\xi} \frac{d\mathcal{P}_{A \rightarrow B+C} \left(\frac{z}{\xi}, \mu^2\right)}{dzdk_T^2} f_A(\xi, \mu^2)
\]

- Final state radiation (FSR): Fragmentations (parton showers):

\[
\Delta_A(t) = \exp \left[ -\sum_B \int_{t_0}^{t} dz \mathcal{P}_{A \rightarrow B+C}(z) \right],
\]

\[
f_A(x, t) = \Delta_A(t)f_A(x, t_0) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \mathcal{P}_{A \rightarrow B+C}(z)f_A(x/z, t')
\]

- Very important formulation for the LHC physics, and future colliders.