Parton contents of a lepton at high energies

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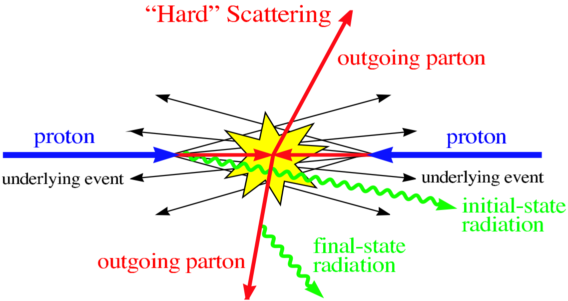
[T. Han, Y. Ma, K.Xie 2007.14300]

[T. Han, Y. Ma, K.Xie 2103.09844]

A little bit background (I): What is "parton"?

Recall the hadron colliders: pp collision at the Tevatron or the LHC



Factorization formalism: PDFs $\otimes$ partonic cross sections

$$\sigma(AB \to X) = \sum_{a,b} \int \mathrm{d}x_a \mathrm{d}x_b f_{a/A}(x_a, Q) f_{b/B}(x_b, Q) \hat{\sigma}(ab \to X)$$

- a, b are the "partons" from the beam particles A and B.
- $f_{a/A}$, $f_{b/B}$ are the probabilities to find a parton a (b) from the beam particle A (B) with a momentum fraction x_a (x_b).

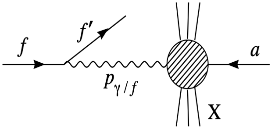
A little bit backaground (II): PDFs of a lepton

"Equivalent photon approximation (EPA)" [C. F. von Weizsacker, Z. Phys. 88, 612 (1934)]

Treat photon as a parton constituent in the electron [E. J. Williams, Phys. Rev. 45, 729 (1934)]

$$\sigma(\ell^- + a \to \ell^- + X) = \int \mathrm{d}x\, f_{\gamma/\ell}\hat{\sigma}(\gamma a \to X)$$

$$f_{\gamma/\ell,\mathrm{EPA}}(x_\gamma, Q^2) = \frac{\alpha}{2\pi}\frac{1+(1-x_\gamma)^2}{x_\gamma}\ln\frac{Q^2}{m_\ell^2}$$



Extra terms:

[Frixione, Mangano, Nason, Ridolfi 2103.09844]
[Budnev, Ginzburg, Meledin, Serbo, Phys. Rept.(1975)]

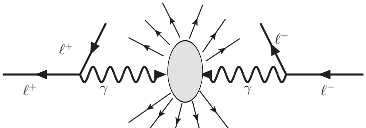
Applications at lepton colliders

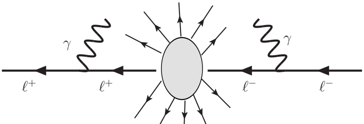
- Production cross sections

$$\sigma(\ell^+\ell^- \to F + X) = \int_{\tau_0}^{1} d\tau \sum_{ij} \frac{d\mathscr{L}_{ij}}{d\tau}\ \hat{\sigma}(ij \to F),\ \tau = \hat{s}/s$$

- Partonic luminosities

$$\frac{d\mathscr{L}_{ij}}{d\tau} = \frac{1}{1+\delta_{ij}}\int_{\tau}^{1}\frac{d\xi}{\xi}\left[f_i(\xi, Q^2) f_j\left(\frac{\tau}{\xi}, Q^2\right) + (i \leftrightarrow j)\right]$$

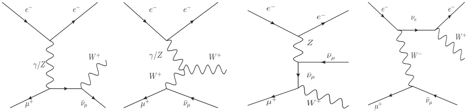




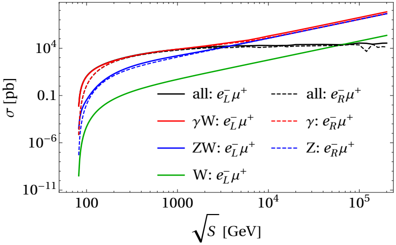
An anatomical study: Why EWPDF?

Fixed order (FO) VS EPA for $e^- \mu^+ \to e^- W^+ \overline{\nu}_\mu$

- Feynman diagrams

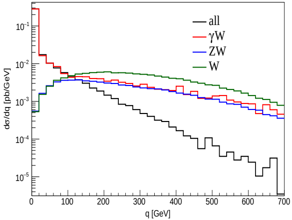


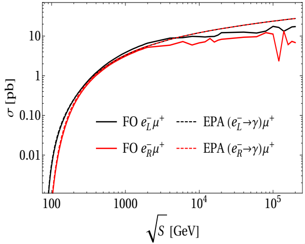
- Q: Is it safe to consider only photon ?
- Things to know:
  - Gauge invariance requires all diagrams
  - Unphysical results appear at high energies



- Need more PDFs, we should go beyond EPA !

- **Q: Why EPA works?**
- **A: Forward γ dominates**

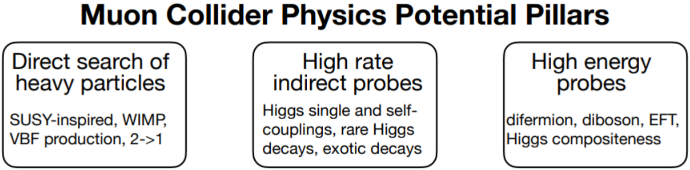




A possible high-energy lepton (muon) collider

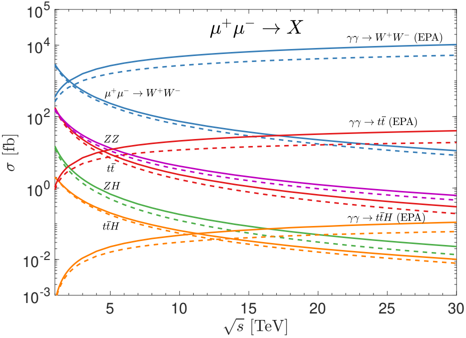
Why muon colliders?

- **Leptons** are the ideal probes of short-distance physics
  - Nearly all the energy is stored in the colliding partons
  - High-energy physics probed with much smaller collider energy
- **Electrons** radiate too much
- A s-channel Higgs factory: Direct measurements on y_μ and Γ_H
- Multi-TeV muon colliders:
  - Center of mass energy $3-15$ TeV and the more speculative $E_{\rm cm}=30$ TeV
  - Luminosity: $\mathscr{L}=(E_{\rm cm}/10\ \mathrm{TeV})^2\times 10\mathrm{ab}^{-1}$
  - New particle mass coverage $M\sim(0.5-1)E_{\rm cm}$
  - Great accuracies for WWH, $WWHH$, H^3, H^4
  - $\cdots$



A high-energy lepton collider at first glance (I)

What do people expect from a high-energy lepton collider?



[T. Han, Y. Ma, K.Xie 2007.14300]

Some "commonsense":

- The annihilations decrease as $1/s$.
- ISR needs to be considered, which can give over 10% enhancement.
- The fusions increase as $\ln^p(s)$, which take over at high energies.
- The large collinear logarithm $\ln\left(s/m_\ell^2\right)$ needs to be resummed, set $Q = \sqrt{\hat{s}}/2$,
- $\gamma\gamma \to W^+W^-$ production has the largest cross section.

A high-energy lepton collider at first glance (II)

What are the dominant processes at a high-energy lepton collider?

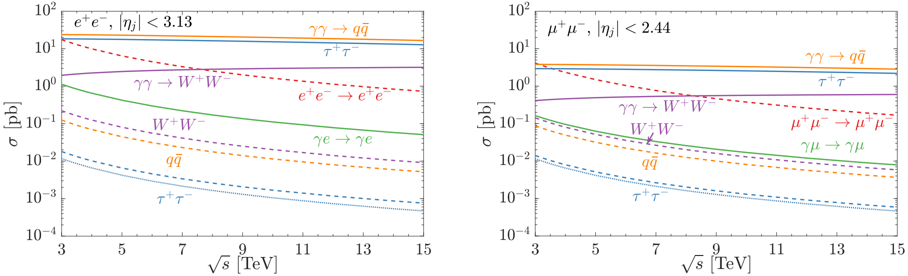
- Leading-order: $\ell^+\ell^- \to \ell^+\ell^-$, $\tau^+\tau^-$, $q\bar{q}$, W^+W^-, and $\gamma\ell \to \gamma\ell$
- $\gamma\gamma$ scatterings: $\gamma\gamma \to \tau^+\tau^-$, $q\bar{q}$, W^+W^-

Need some cuts:

- Detector angle: $\theta_{\rm cut} = 5^\circ\,(10^\circ) \Longleftrightarrow |\eta| < 3.13(2.44)$
- Threshold: $m_{ij} > 20$ GeV
- Need a p_T cut to separate from the nonperturbative hadronic production

[Drees and Godbole, PRL 67, 1189; Chen, Barklow, and Peskin, hep-ph/9305247;T. Barklow, etal, LCD-2011-020]

$$p_T > \left(4 + \sqrt{s}/3\,\mathrm{TeV}\right)\,\mathrm{GeV}$$

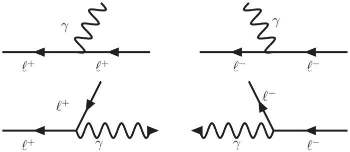


Go beyond the EPA at high-energy lepton colliders

We have been doing:

- $\ell^+\ell^-$ annihilation

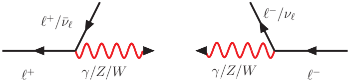
- EPA and ISR



- "Effective W Approx." (EWA)

  [G. Kane, W. Repko, and W. Rolnick, PLB 148 (1984) 367]

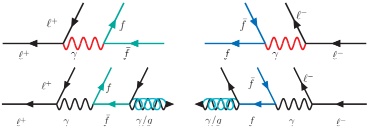
  [S. Dawson, NPB 249 (1985) 42]



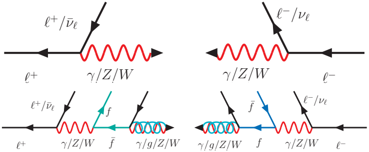
We will add:

[T. Han, Y. Ma, K.Xie 2007.14300, 2103.09844]

- Above $\mu_{\rm QCD}$: QED⊗QCD
  q/g emerge



- Above $\mu_{\rm EW} = M_Z$: EW⊗QCD
  EW partons emerge



In the end, everything is parton, i.e. the full SM PDFs.

The PDF evolution: DGLAP

- The DGLAP equations

$$\frac{\mathrm{d}f_i}{\mathrm{d}\log Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P_{ij}^I \otimes f_j$$

- The initial conditions

$$f_{\ell/\ell}(x, m_\ell^2) = \delta(1-x)$$

- Three regions and two matchings
  - $m_\ell < Q < \mu_{\rm QCD}$: QED
  - $Q = \mu_{\rm QCD} \lesssim 1$ GeV: $f_q \propto P_{q\gamma} \otimes f_\gamma, f_g = 0$
  - $\mu_{\rm QCD} < Q < \mu_{\rm EW}$: QED$\otimes$QCD
  - $Q = \mu_{\rm EW} = M_Z$: $f_\nu = f_t = f_W = f_Z = f_{\gamma Z} = 0$
  - $\mu_{\rm EW} < Q$: EW$\otimes$QCD.

$$\begin{pmatrix} f_B \\ f_{W^3} \\ f_{BW^3} \end{pmatrix} = \begin{pmatrix} c_W^2 & s_W^2 & -2c_W s_W \\ s_W^2 & c_W^2 & 2c_W s_W \\ c_W s_W & -c_W s_W & c_W^2 - s_W^2 \end{pmatrix} \begin{pmatrix} f_\gamma \\ f_Z \\ f_{\gamma Z} \end{pmatrix}$$

- We work in the (B, W) basis. The technical details can be referred to the backup slides.

Solving the DGLAP: Singlet and Non-singlet PDFs

The singlets

$$f_L = \sum_{i=e,\mu,\tau}(f_{\ell_i} + f_{\bar{\ell}_i}),\ f_U = \sum_{i=u,c}(f_{u_i} + f_{\bar{u}_i}),\ f_D = \sum_{i=d,s,b}(f_{d_i} + f_{\bar{d}_i})$$

The non-singlets

- The only non-trivial singlet $f_{e,NS} = f_e - f_{\bar{e}}$
- the leptons

$$f_{\ell_i,NS} = f_{\ell_i} - f_{\bar{\ell}_i}(i=2,3),\ f_{\ell,12} = f_{\bar{e}} - f_{\bar{\mu}},\ f_{\ell,13} = f_{\bar{e}} - f_{\bar{\tau}};$$

- the up-type quarks

$$f_{u_i,NS} = f_{u_i} - f_{\bar{u}_i},\ f_{u,12} = f_u - f_c;$$

- and the down-type quarks

$$f_{d_i,NS} = f_{d_i} - f_{\bar{d}_i},\ f_{d,12} = f_d - f_s,\ f_{d,13} = f_d - f_b.$$

Reconstruction:

$$f_e = \frac{f_L + (2N_\ell - 1)f_{e,NS}}{2N_\ell},\ f_{\bar{e}} = f_\mu = f_{\bar{\mu}} = f_\tau = f_{\bar{\tau}} = \frac{f_L - f_{e,NS}}{2N_\ell}.$$

$$f_u = f_{\bar{u}} = f_c = f_{\bar{c}} = \frac{f_U}{2N_u},\ f_d = f_{\bar{d}} = f_s = f_{\bar{s}} = f_b = f_{\bar{b}} = \frac{f_D}{2N_d}.$$

The QED⊗QCD case

- The singlets and gauge bosons

$$\frac{\mathrm{d}}{\mathrm{d}\log Q^2}\begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix} = \begin{pmatrix} P_{\ell\ell} & 0 & 0 & 2N_\ell P_{\ell\gamma} & 0 \\ 0 & P_{uu} & 0 & 2N_u P_{u\gamma} & 2N_u P_{ug} \\ 0 & 0 & P_{dd} & 2N_d P_{d\gamma} & 2N_d P_{dg} \\ P_{\gamma\ell} & P_{\gamma u} & P_{\gamma d} & P_{\gamma\gamma} & 0 \\ 0 & P_{gu} & P_{gd} & 0 & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix}$$

- The non-singlets

$$\frac{\mathrm{d}}{\mathrm{d}\log Q^2} f_{NS} = P_{ff} \otimes f_{NS}.$$

- The averaged momentum fractions of the PDFs: $f_{\ell_{\mathrm{val}}}, f_\gamma, f_{\ell_{\mathrm{sea}}}, f_q, f_g$

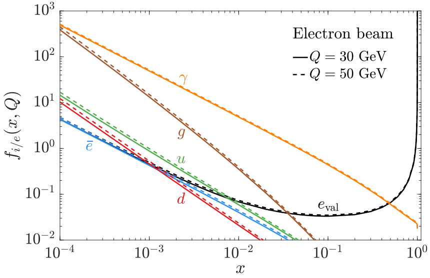
$$\langle x_i \rangle = \int x f_i(x)\mathrm{d}x,\ \sum_i \langle x_i \rangle = 1$$

$$\frac{\langle x_q \rangle}{\langle x_{\ell\mathrm{sea}} \rangle} \lesssim \frac{N_c\left[\sum_i (e_{u_i}^2 + e_{\bar{u}_i}^2) + \sum_i (e_{d_i}^2 + e_{\bar{d}_i}^2)\right]}{e_{\bar{\ell}_{\mathrm{val}}}^2 + \sum_{i\neq \ell\mathrm{val}} (e_{\ell_i}^2 + e_{\bar{\ell}_i}^2)} = \frac{22/3}{5}$$

The QED⊗QCD PDFs for lepton colliders

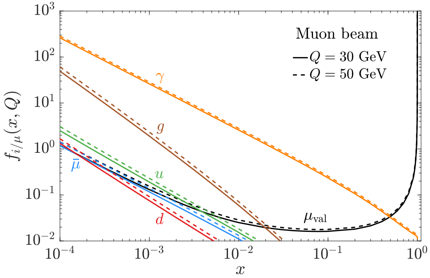
- **Electron PDFs**: $f_{e_{\rm val}}, f_\gamma, f_{\ell_{\rm sea}}, f_q, f_g$
- Scale uncertainty: 10% for $f_{g/e}$
- The averaged momentum fractions $\langle x_i \rangle = \int x f_i(x) \mathrm{d}x$

$Q(e^\pm)$	$e_{\rm val}$	γ	ℓsea	q	g
30 GeV	96.6	3.20	0.069	0.080	0.023
50 GeV	96.5	3.34	0.077	0.087	0.026
M_Z	96.3	3.51	0.085	0.097	0.028



- **Muon PDFs**: $f_{\mu_{\rm val}}, f_\gamma, f_{\ell_{\rm sea}}, f_q, f_g$
- Scale uncertainty: 20% for $f_{g/\mu}$
- The averaged momentum fractions $\langle x_i \rangle = \int x f_i(x) \mathrm{d}x$

$Q(\mu^\pm)$	$\mu_{\rm val}$	γ	ℓsea	q	g
30 GeV	98.2	1.72	0.019	0.024	0.0043
50 GeV	98.0	1.87	0.023	0.029	0.0051
M_Z	97.9	2.06	0.028	0.035	0.0062



The DGLAP for the full SM

$$\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix} f_L^{0\pm} \\ f_Q^{0\pm} \\ f_E^{0\pm} \\ f_U^{0\pm} \\ f_D^{0\pm} \\ f_B^{0\pm} \\ f_W^{0\pm} \\ f_g^{0\pm} \end{pmatrix} = \begin{pmatrix} P_{LL}^{0\pm} & 0 & 0 & 0 & 0 & P_{LB}^{0\pm} & P_{LW}^{0\pm} & 0 \\ 0 & P_{QQ}^{0\pm} & 0 & 0 & 0 & P_{QB}^{0\pm} & P_{QW}^{0\pm} & P_{Qg}^{0\pm} \\ 0 & 0 & P_{EE}^{0\pm} & 0 & 0 & P_{EB}^{0\pm} & 0 & 0 \\ 0 & 0 & 0 & P_{UU}^{0\pm} & 0 & P_{UB}^{0\pm} & 0 & P_{Ug}^{0\pm} \\ 0 & 0 & 0 & 0 & P_{DD}^{0\pm} & P_{DB}^{0\pm} & 0 & P_{Dg}^{0\pm} \\ P_{BL}^{0\pm} & P_{BQ}^{0\pm} & P_{BE}^{0\pm} & P_{BU}^{0\pm} & P_{BD}^{0\pm} & P_{BB}^{0\pm} & 0 & 0 \\ P_{WL}^{0\pm} & P_{WQ}^{0\pm} & 0 & 0 & 0 & 0 & P_{WW}^{0\pm} & 0 \\ 0 & P_{gQ}^{0\pm} & 0 & P_{gU}^{0\pm} & P_{gD}^{0\pm} & 0 & 0 & P_{gg}^{0\pm} \end{pmatrix} \otimes \begin{pmatrix} f_L^{0\pm} \\ f_Q^{0\pm} \\ f_E^{0\pm} \\ f_U^{0\pm} \\ f_D^{0\pm} \\ f_B^{0\pm} \\ f_W^{0\pm} \\ f_g^{0\pm} \end{pmatrix}$$

$$\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix} f_L^{1\pm} \\ f_Q^{1\pm} \\ f_W^{1\pm} \\ f_{BW}^{1\pm} \end{pmatrix} = \begin{pmatrix} P_{LL}^{1\pm} & 0 & P_{LW}^{1\pm} & P_{LM}^{1\pm} \\ 0 & P_{QQ}^{1\pm} & P_{QW}^{1\pm} & P_{QM}^{1\pm} \\ P_{WL}^{1\pm} & P_{WQ}^{1\pm} & P_{WW}^{1\pm} & 0 \\ P_{ML}^{1\pm} & P_{MQ}^{1\pm} & 0 & P_{MM}^{1\pm} \end{pmatrix} \otimes \begin{pmatrix} f_L^{1\pm} \\ f_Q^{1\pm} \\ f_W^{1\pm} \\ f_{BW}^{1\pm} \end{pmatrix}$$

$$\frac{\mathrm{d}}{\mathrm{d}L} f_W^{2\pm} = P_{WW}^{2\pm} \otimes f_{WW}^{2\pm}$$

The splitting functions can be constructed in terms of Refs.

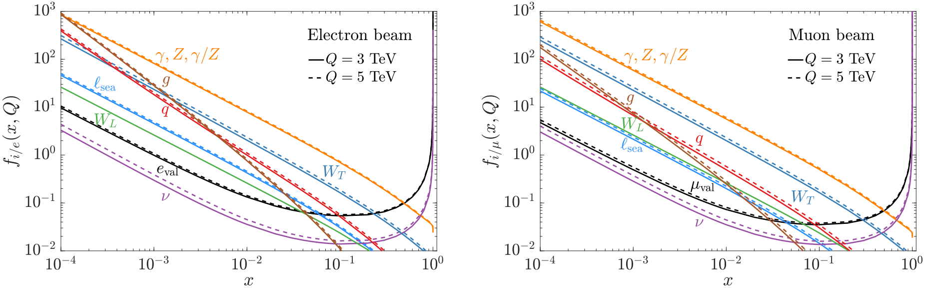
[Chen et al. 1611.00788, Bauer et al. 1703.08562,1808.08831]

EWPDFs of a lepton

- The sea leptonic and quark PDFs

$$\nu = \sum_i (\nu_i + \bar{\nu}_i),\ \ \ell\text{sea} = \bar{\mu} + \sum_{i \neq \mu} (\ell_i + \bar{\ell}_i),\ \ q = \sum_{i=d}^{t} (q_i + \bar{q}_i)$$

There is even neutrino due to the EW sector, **everything is a parton!**



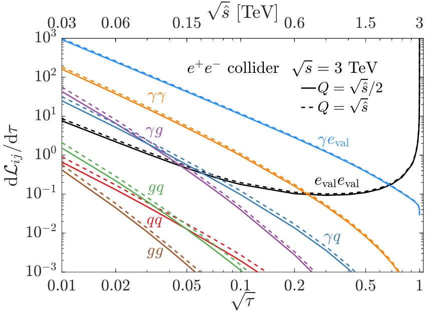
- **All SM particles are partons** [T. Han, Y. Ma, K.Xie 2007.14300]
- W_L does not evolve: **Bjorken-scaling restoration**.
- The EW correction is not small: $\sim 50\%$ (100%) for $f_{d/e}$ $(f_{d/\mu})$ due to the relatively **large SU(2) gauge coupling**. [T. Han, Y. Ma, K.Xie 2103.09844]
- Scale uncertainty: $\sim 15\%$ (20%) between $Q = 3$ TeV and $Q = 5$ TeV

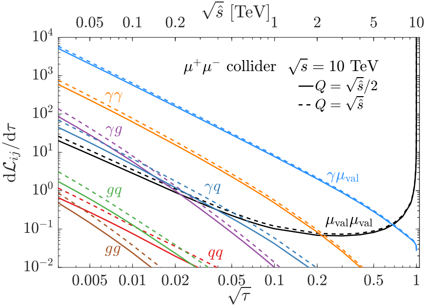
Parton luminosities at high-energy lepton colliders

Consider a 3 TeV e^+e^- machine and a 10 TeV $\mu^+\mu^-$ machine

- Partonic luminosities for

$$\ell^+\ell^-, \gamma\ell, \gamma\gamma, qq, \gamma q, \gamma g, gq, \text{ and } gg$$





- The partonic luminosity of $\gamma g + \gamma q$ is $\sim 50\%$ (20%) of the $\gamma\gamma$ one
- The partonic luminosities of qq, gq, and gg are $\sim 2\%$ (0.5%) of the $\gamma\gamma$ one
- Given the stronger QCD coupling, **sizable QCD cross sections are expected**.
- Scale uncertainty is $\sim 20\%$ (50%) for photon (gluon) initiated processes.

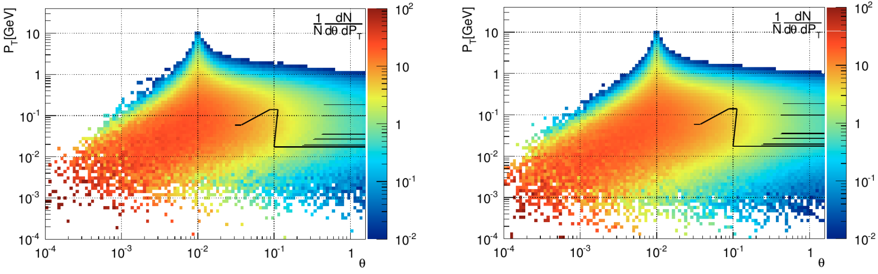
Jet production of possible lepton colliders (I)

- Large photon induced non-perturbative hadronic production

  [Drees and Godbole, PRL 67 1189, hep-ph/9203219]

  [Chen, Barklow, and Peskin, hep-ph/9305247; Godbole, Grau, Mohan, Pancheri, SrivastavaNuovo Cim. C 034S1]

  - $\sigma_{\gamma\gamma}$ may reach micro-barns level at TeV c.m. energies
  - $\sigma_{\ell\ell}$ may reach nano-barns, after folding in the $\gamma\gamma$ luminosity
- The events populate at low p_T regime
  So we can separate from this non-perturbative range via a p_T cut.



(a) Pythia sample

(b) SLAC sample

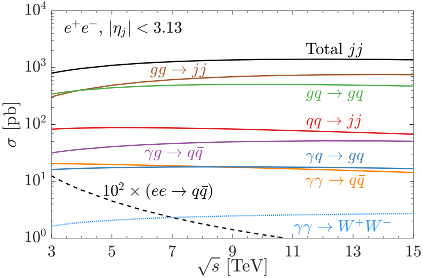
[T. Barklow, D. Dannheim, M. O. Sahin, and D. Schulte, LCD-2011-020]

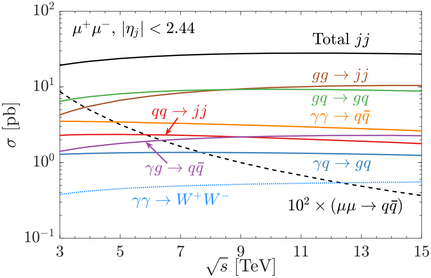
Jet production at possible lepton colliders (II)

- High-p_T range $[p_T > (4+\sqrt{s}/3\,\mathrm{TeV})\ \mathrm{GeV}]$: perturbatively computable

$$\gamma\gamma \to q\bar{q},\ \gamma g \to q\bar{q},\ \gamma q \to gq,$$
$$qq \to qq\ (gg),\ gq \to gq \text{ and } gg \to gg\ (q\bar{q}).$$

- Large $\alpha_s \ln(Q^2)$ brings a $6\% \sim 15\%$ $(30\% \sim 40\%)$ enhancement if $Q = 2Q$





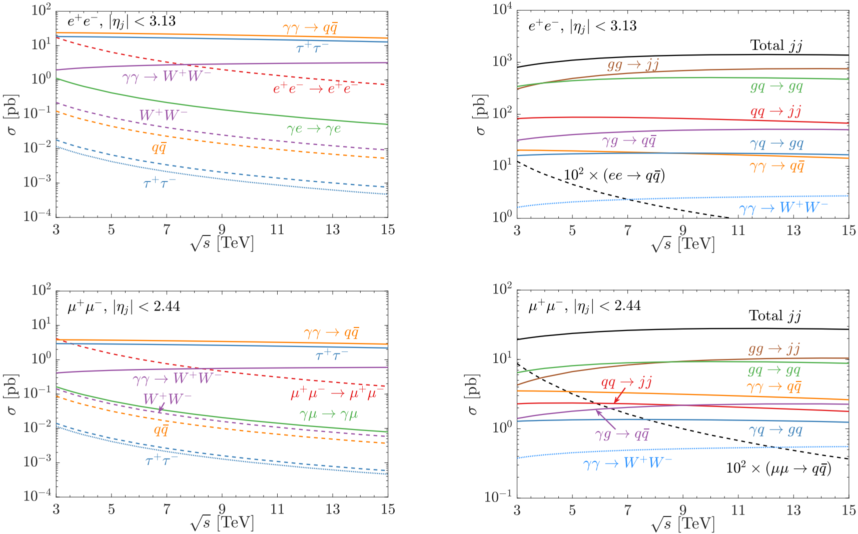
- Including the QCD contribution leads to much larger total cross section.
- gg initiated cross sections are large for its large multiplicity;
- gq initiated cross sections are large for its large luminosity.
- $\gamma\gamma$ initiated cross sections here are smaller than the EPA results.

Refresh the picture of high-energy lepton colliders

What is the dominant process at a high-energy muon collider?

- **Quark/gluon initiated jet production dominates**

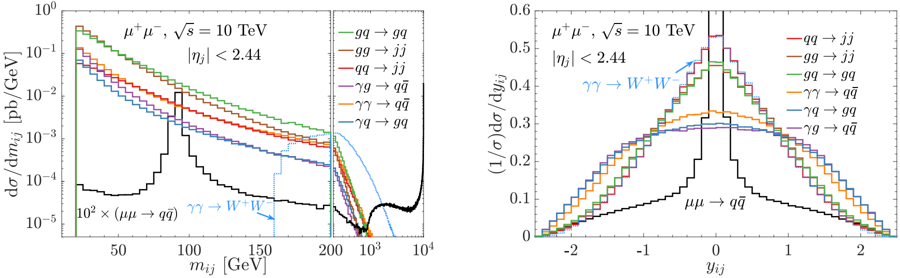
Before: **After:**



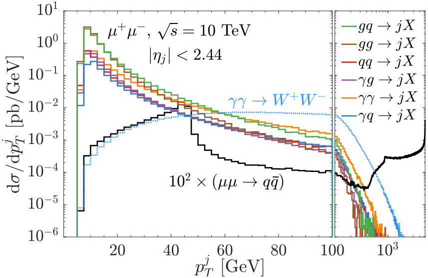
Di-jet distributions at a muon collider

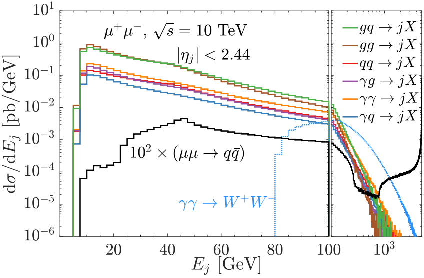
Rather a conservative set up: $\theta = 10^\circ$

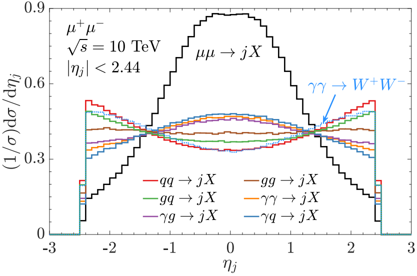
- Some physics:
  Two different mechanisms: $\mu^+\mu^-$ **annihilation** VS **Fusion processes**
  - Annihilation is more than 2 orders of magnitude smaller than fusion process.
  - Annihilation peaks at $m_{ij} \sim \sqrt{s}$;
  - Fusion processes peak near m_{ij} threshold.
  - Annihilation is very central, spread out due to ISR;
  - Fusion processes spread out, especially for γq and γg initiated ones.



Inclusive jet distributions at a muon collider







- Jet production dominates over WW production until $p_T > 60$ GeV;
- WW production takes over around energy ~ 200 GeV.
- QCD contributions are mostly forward-backward; $\gamma\gamma$, γq, and γg initiated processes are more isotropic.

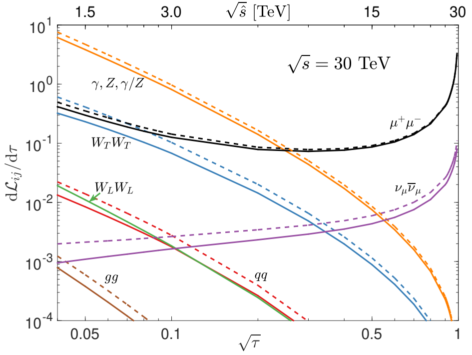
The EW parton luminosities of a 30 TeV muon collider

- Production cross sections

$$\sigma(\ell^+\ell^- \to F + X) = \int_{\tau_0}^{1} d\tau \sum_{ij} \frac{d\mathscr{L}_{ij}}{d\tau}\ \hat{\sigma}(ij \to F),\ \ \tau = \hat{s}/s$$

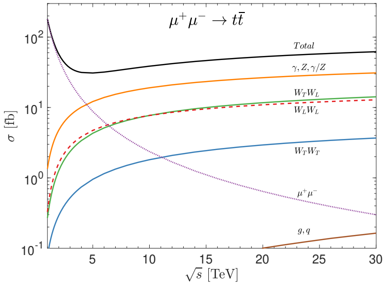
- Partonic luminosities

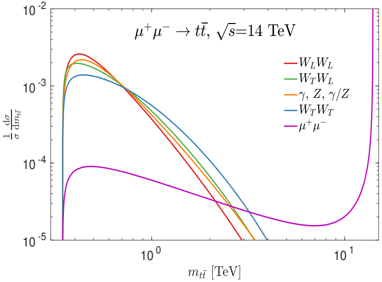
$$\frac{d\mathscr{L}_{ij}}{d\tau} = \frac{1}{1+\delta_{ij}} \int_{\tau}^{1} \frac{d\xi}{\xi} \left[f_i(\xi, Q^2) f_j\left(\frac{\tau}{\xi}, Q^2\right) + (i \leftrightarrow j) \right]$$

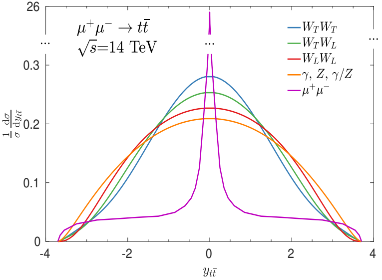


[T. Han, Y. Ma, K.Xie 2007.14300]

One example: $t\bar{t}$ production at a muon collider

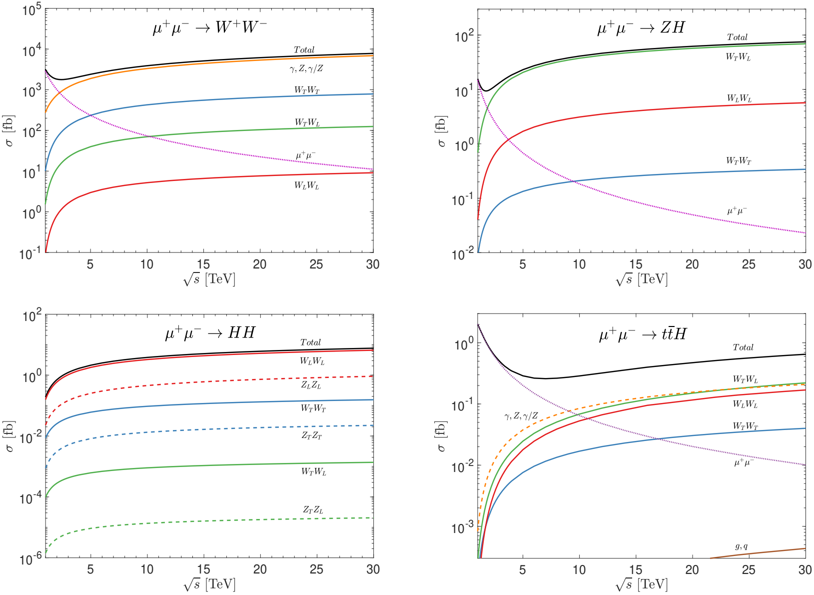






[T. Han, Y. Ma, K.Xie 2007.14300]

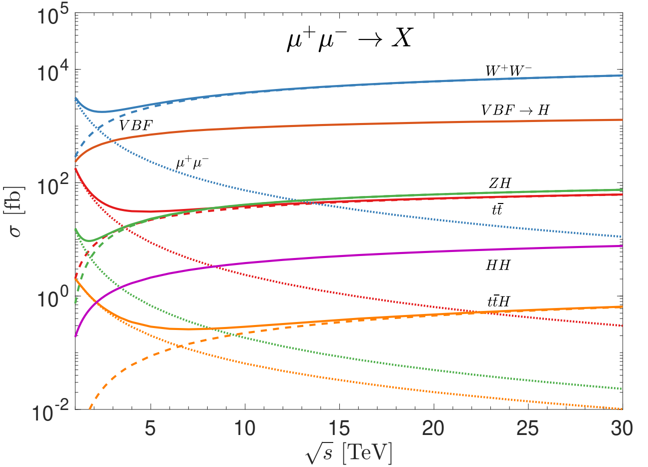
Other processes: $W^+W^-, ZH, HH, t\bar{t}H$



The full picture: Semi-inclusive processes

Just like in hadronic collisions:

$$\mu^+\mu^- \to \text{exclusive particles} + \text{remnants}$$



[T. Han, Y. Ma, K.Xie 2007.14300]

Summary and prospects

EWPDF:

- At very high energies, the collinear splittings dominate. **All SM particles should be treated as partons that described by proper PDFs.**
  - The large collinear logarithm needs to be resummed via solving the DGLAP equations, so the **QCD partons (quarks and gluons) emerge**.
  - When $Q > M_Z$, the EW splittings are activated: the EW partons appear, and the existing QED$\otimes$QCD PDFs may receive big corrections.

For a high-energy lepton (muon) collider:

- There are many things to work on: SUSY, DM, Higgs, etc.
- The parton fusion processes exceed the $\ell^+\ell^-$ annihilation, where we should employ the EWPDFs [T. Han, Y. Ma, K.Xie 2007.14300]
- **The main background of is the jet production**:
  - Low p_T range: non-perturbative $\gamma\gamma$ initiated hadronic production dominates [Chen, Barklow, and Peskin, hep-ph/9305247; Drees and Godbole, PRL 67, 1189,T. Barklow, etal, LCD-2011-020]
  - High p_T range, q and g initiated jet production dominates [T. Han, Y. Ma, K.Xie 2103.09844]
- EWPDF allows to determine the contributions from different partons and their different ploarizations
- One should look at the semi-inclusive processes

The novel features of the EW PDFs

- The EW PDFs must be polarized due to the chiral nature of the EW theory

$$f_{V_+/A_+} \neq f_{V_-/A_-}, \qquad f_{V_+/A_-} \neq f_{V_-/A_+},$$
$$\hat{\sigma}(V_+B_+) \neq \hat{\sigma}(V_-B_-), \quad \hat{\sigma}(V_+B_-) \neq \hat{\sigma}(V_-B_+)$$

We are not able to factorize the cross sections in an unporlarized form.

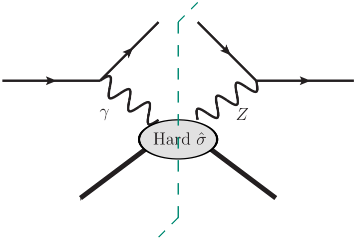
$$\sigma \neq f_{V/A}\hat{\sigma}(VB),\ f_{V/A} = \frac{1}{2}\sum_{\lambda,s_1} f_{V_\lambda/A_{s_1}},\ \hat{\sigma}(VB) = \frac{1}{4}\sum_{\lambda,s_2}\hat{\sigma}(V_\lambda B_{s_2})$$

- The interference gives the mixed PDFs

[Bauer '17, '18, Manohar '18 , TH '16.]

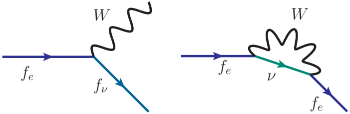
$$f_{\gamma Z} \sim \langle \Omega | A^{\mu\nu} Z_{\mu\nu} | \Omega \rangle + \text{h.c.},$$

similarly for f_{hZ_L}.



- Bloch-Nordsieck theorem violation due to the non-cancelled divergence in $f \to f' V$: cutoff M_V/Q or redefinition

$$f_1 \sim f_e + f_\nu,\ f_3 \sim f_e - f_\nu$$



Return to the begining: $e^- \mu^+ \to e^- W^+ \bar{\nu}_\mu$

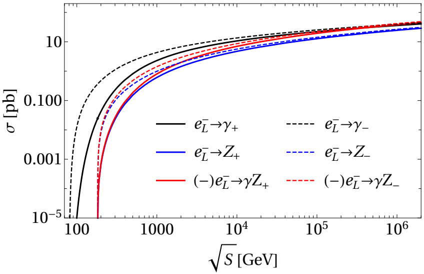
- EWA: $f_{V_\lambda/e_s^\pm}(x,Q) = \frac{1}{8\pi^2} g_1 g_2 P_{V_\lambda/e_s^\pm}(x) \log\left(Q^2/m_Z^2\right)$

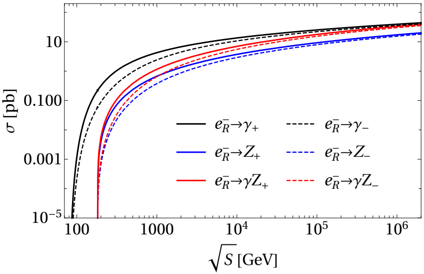
  $g_L = \frac{g_2}{c_W}\left(-\frac{1}{2}+s_W^2\right) < 0,\ g_R = \frac{g_2}{c_W} s_W^2 > 0,\ g_e = -e$

	e_L^-	e_R^-	e_L^+	e_R^+
Z_-	$g_L^2 \frac{1}{x}$	$g_R^2 \frac{(1-x)^2}{x}$	$g_L^2 \frac{(1-x)^2}{x}$	$g_R^2 \frac{1}{x}$
Z_+	$g_L^2 \frac{(1-x)^2}{x}$	$g_R^2 \frac{1}{x}$	$g_L^2 \frac{1}{x}$	$g_R^2 \frac{(1-x)^2}{x}$
γZ_-	$g_e g_L \frac{1}{x}$	$g_e g_R \frac{(1-x)^2}{x}$	$g_e g_L \frac{(1-x)^2}{x}$	$g_e g_R \frac{1}{x}$
γZ_+	$g_e g_L \frac{(1-x)^2}{x}$	$g_e g_R \frac{1}{x}$	$g_e g_L \frac{1}{x}$	$g_e g_R \frac{(1-x)^2}{x}$

- The contribution of the mixed PDF $f_{\gamma Z}$ can be either constructive or destructive

$$\sigma = \sum_{\lambda, s_1, s_2} f_{V_\lambda/e_{s_1}^-} \hat{\sigma}(V_\lambda \mu_{s_2}^+ \to W^+ \bar{\nu}_\mu)$$

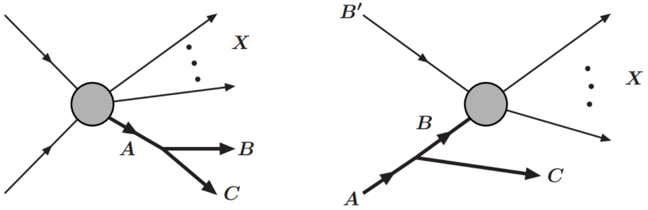




EW physics at high energies

- At high energies, every particle become massless
$$\frac{v}{E} : \frac{v}{100\ \text{TeV}} \sim \frac{\Lambda_{\text{QCD}}}{100\ \text{GeV}},\ \frac{v}{E}, \frac{m_t}{E}, \frac{M_W}{E} \to 0!$$
- The splitting phenomena dominate due to large log enhancement
- The EW symmetry is restored: $SU(2)_L \times U(1)_Y$ unbroken
- Goldstone Boson Equivalence:
$$\varepsilon_L^\mu(k) = \frac{E}{M_W}(\beta_W, \hat{k}) \simeq \frac{k^\mu}{M_W} + \mathscr{O}(\frac{M_W}{E})$$
The violation terms is power counted as $v/E \to$ QCD higher twist effects Λ_{QCD}/Q [G. Cuomo, A. Wulzer, arXiv:1703.08562; 1911.12366].
- We mainly focus on the splitting phenomena, which can be factorized and resummed as the EW PDFs in the ISR, and the Fragementaions/Parton Shower in the FRS.
- Other interesting aspects: the polarized EW boson scattering, top-Yukawa coupling effect

Factorization of the EW splittings



$$\mathrm{d}\sigma \simeq \mathrm{d}\sigma_X \times \mathrm{d}\mathscr{P}_{A\to B+C}\,, \quad E_B \approx zE_A, \quad E_C \approx \bar{z}E_A, \quad k_T \approx z\bar{z}E_A\theta_{BC}$$

$$\frac{\mathrm{d}\mathscr{P}_{A\to B+C}}{\mathrm{d}z\mathrm{d}k_T^2} \simeq \frac{1}{16\pi^2}\frac{z\bar{z}|\mathscr{M}^{(\mathrm{split})}|^2}{(k_T^2+\bar{z}m_B^2+zm_C^2-z\bar{z}m_A^2)^2}, \quad \bar{z}=1-z$$

- The dimensional counting: $|\mathscr{M}^{(\mathrm{split})}|^2 \sim k_T^2$ or m^2
- To validate the fractorization formalism
  - The observable σ should be infra-red safe
  - Leading behavior comes from the collinear splitting

[Ciafaloni et al., hep-ph/0004071; 0007096; C. Bauer, Ferland, B. Webber et al., arXiv:1703.08562;1808.08831]

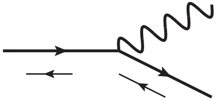
[A. Manohar et al., 1803.06347; T. Han, J. Chen & B. Tweedie, arXiv:1611.00788]

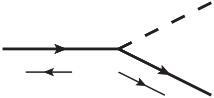
Splitting functions: EW

- Starting from the unbroken phase: all massless

$$\mathscr{L}_{SU(2)\times U(1)} = \mathscr{L}_{gauge} + \mathscr{L}_{\phi} + \mathscr{L}_{f} + \mathscr{L}_{Yukawa}$$

- Particle contents:
  - Chiral fermions $f_{L,R}$
  - Gauge bosons: $B, W^{0,\pm}$
  - Higgs $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(h - i\phi^0) \end{pmatrix}$
- Splitting functions [See Ciafaloni et al. hep-ph/0505047, Chen et al. 1611.00788 for complete lists.]

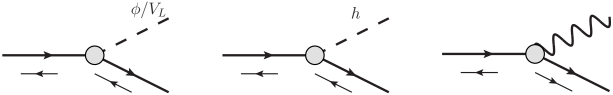




	$\frac{1}{8\pi^2}\frac{1}{k_T^2}\frac{1+\bar{z}^2}{z}$		$\frac{1}{8\pi^2}\frac{1}{k_T^2}\frac{z}{2}$	
$f_{s=L,R} \to$	$V_T f_s^{(\prime)}$	$[BW]^0_T f_s$	$H^{0*} f_{-s}$	$\phi^{\pm} f'_{-s}$
	$g_V^2 (Q^V_{f_s})^2$	$g_1 g_2 Y_{f_s} T^3_{f_s}$	$y^2_{f_R^{(\prime)}}$	
	Infrared, collinear singularites (P_{gq})		Collinear singularity chirality-flip, Yukawa	

Corrections to the GET in the EWSB

- New fermion splitting: $P \sim \frac{v^2}{k_T^2}\frac{\mathrm{d}k_T^2}{k_T^2}$
- V_L is of IR, h has no IR



	$\frac{1}{16\pi^2}\frac{v^2}{k_T^4}\frac{1}{z}$	$\frac{1}{16\pi^2}\frac{v^2}{k_T^4}$	$\frac{1}{16\pi^2}\frac{v^2}{k_T^4}$
$f_s \to$	$V_L f_s^{(')} (V \neq \gamma)$	$h f_s$	$V_T f_{-s}^{(')}$
	Chirality conserving non-zero for massless f		Chirality flipping $\sim m_f$

- The PDFs for W_L/Z_L behaves as constants, which does not run at the leading log: "Bjorken scaling" restoration (higher-twist effects)

$$f_{V_L/f}(x, Q^2) \sim \alpha \frac{1-x}{x}$$

PDFs and Fragmentations (parton showers)

- Initial state radiation (ISR), PDFs (DGLAP):

$$f_B(z,\mu^2) = \sum_A \int_z^1 \frac{d\xi}{\xi} f_A(\xi) \int_{m^2}^{\mu^2} d\mathscr{P}_{A\to B+C}(z/\xi, k_T^2)$$

$$\frac{\partial f_B(z,\mu^2)}{\partial \mu^2} = \sum_A \int_z^1 \frac{d\xi}{\xi} \frac{d\mathscr{P}_{A\to B+C}(z/\xi,\mu^2)}{dz dk_T^2} f_A(\xi,\mu^2)$$

- Final state radiation (FSR): Fragmentations (parton showers):

$$\Delta_A(t) = \exp\left[-\sum_B \int_{t_0}^t \int dz \mathscr{P}_{A\to B+C}(z)\right],$$

$$f_A(x,t) = \Delta_A(t) f_A(x,t_0) + \int_{t_0}^t \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \mathscr{P}_{A\to B+C}(z) f_A(x/z,t')$$

- Very important formulation for the LHC physics, and future colliders.