The on-shell SM EFTs

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JHEP 01 (2020) 119, [`1909.10551`]
with Teppei Kitahara, Yael Shadmi, Yaniv Weiss

Phys. Rev. D101 (2020) 095021, [`1912.08827`]
with Camila Machado

JHEP 12 (2020) 175, [`2008.09652`]
with Teppei Kitahara, Camila Machado, Yael Shadmi, Yaniv Weiss

Oklahoma State HEP Seminar, 23 Sep 2021

Lagrangians are powerful... but not always

Fields are unphysical and redundant

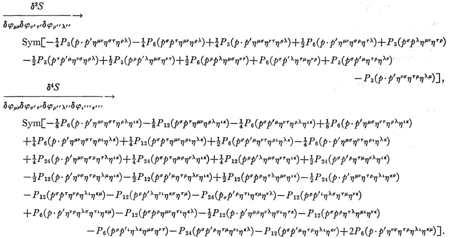
- redefinitions do not alter the physics
- four-vector/tensor to embed two dofs

Lagrangians are powerful... but not always

Fields are u

- redefiniti
- four-vect

graviton Feynman rules [De Witt '67]



$$\frac{\delta^3 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}}\longrightarrow$$

$$\begin{aligned}&\mathrm{Sym}[-\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda})-\tfrac{1}{4}P_6(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda})+\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda})+\tfrac{1}{2}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda})+P_3(p^\sigma p^\lambda\eta^{\mu\nu}\eta^{\tau\rho})\\&-\tfrac{1}{2}P_3(p^\tau p'^\mu\eta^{\nu\sigma}\eta^{\rho\lambda})+\tfrac{1}{2}P_3(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+\tfrac{1}{2}P_6(p^\rho p^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+P_6(p^\sigma p'^\lambda\eta^{\tau\mu}\eta^{\nu\rho})+P_3(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu})\\&\qquad\qquad-P_3(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})],\end{aligned}$$

$$\frac{\delta^4 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}\delta\varphi_{\iota'''\kappa'''}}\longrightarrow$$

$$\begin{aligned}&\mathrm{Sym}[-\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{8}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{4}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})+\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})\\&+\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{4}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{2}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})-\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})\\&+\tfrac{1}{4}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{24}(p^\sigma p^\tau\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{12}(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\iota\kappa})+\tfrac{1}{2}P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa})\\&-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu}\eta^{\iota\kappa})+\tfrac{1}{2}P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\nu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\sigma})\\&-P_{12}(p^\sigma p^\tau\eta^{\nu\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})-P_{12}(p^\rho p'^\lambda\eta^{\nu\iota}\eta^{\kappa\sigma}\eta^{\tau\mu})-P_{24}(p_\sigma p'^\rho\eta^{\tau\iota}\eta^{\kappa\mu}\eta^{\nu\lambda})-P_{12}(p^\rho p'^\iota\eta^{\lambda\sigma}\eta^{\tau\mu}\eta^{\nu\kappa})\\&+P_6(p\cdot p'\eta^{\nu\rho}\eta^{\lambda\sigma}\eta^{\tau\iota}\eta^{\kappa\mu})-P_{12}(p^\sigma p^\rho\eta^{\mu\nu}\eta^{\tau\iota}\eta^{\kappa\lambda})-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\sigma\iota}\eta^{\tau\kappa})-P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\iota}\eta^{\nu\kappa})\\&\qquad\qquad-P_6(p^\rho p'^\iota\eta^{\lambda\kappa}\eta^{\mu\sigma}\eta^{\nu\tau})-P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\iota}\eta^{\kappa\lambda})-P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\nu})+2P_6(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})].\end{aligned}$$

171 & 2850 terms

vs.

$[12]^6/[23]^2[31]^2$ & $[12]^4\langle 34\rangle^4/stu$

On-shell bootstrapping

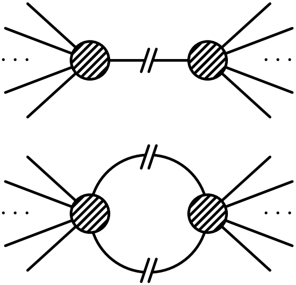
- Trees cut into trees

  \+ contact terms

- Loops cut into trees

  no loops hereafter

factorization/unitarity



Recently

- applications to EFTs
- covariant massive spinor formalism [Arkani-Hamed, Huang, Huang '17]

Contact terms

massless → operator enumeration (unbroken EW symmetry)

massive → bootstrap building blocks

operator enumeration

[Shadmi, Weiss '18], [Ma, Shu, Xiao '19], [Falkowski '19], [GD, Machado '19],
[Li, Ren, et al. '20, '20, '20]

SM(EFT) from massive amplitudes

SM: [Christensen, Field '18], [Bachu, Yelleshpur '19],
SMEFT: [Aoude, Machado '19], [GD, Kitahara, Shadmi, Weiss '19], [GD et al. '20], [Dong, Ma, Shu '21]

EFTs from soft properties

[Cheung et al. '14, '15, '16], [Low et al. '19, '19, '20]

SMEFT non-renormalizations, non-interferences, RGEs

[Cheung, Shen '15], [Azatov et al. '16], [Bern et al. '19, '20],
[Jiang et al. '20], [Elias Miró et al. '20], [Baratella et al. '20, '20],
[Baratella et al. '21], [Accettulli Huber, De Angelis '21]

...

Outline

Bootstrapping EFTs

a. Massless contact terms

b. Massive contact terms

Massive electroweak EFTs

a. Massless contact terms

Helicity spinors

[Mangano, Parke '91]
[Helvang, Huang '13]
[Dixon '13]
[Schwartz '14]
[Cheung '17]

As brackets

$$u_{i^+} = \begin{pmatrix} 0 \\ i] \end{pmatrix}, \qquad u_{i^-} = \begin{pmatrix} i\rangle \\ 0 \end{pmatrix} \qquad \text{for particle } i$$

Rewritting momenta (and polarizations vectors)

$$p_i^\mu \sigma_\mu = i\rangle[i \qquad \left(\varepsilon_{i^+}^\mu \sigma_\mu = \frac{\zeta\rangle[i}{\sqrt{2}\langle \zeta i\rangle}, \qquad \varepsilon_{i^-}^\mu \sigma_\mu = \frac{i\rangle[\zeta}{\sqrt{2}\langle i\zeta\rangle} \right)$$

Trivializing $p_i^2 = \langle ii\rangle[ii]/2 = 0$

$$\langle ii\rangle = \epsilon_{\alpha\beta}\ i\rangle^\alpha i\rangle^\beta = 0, \qquad [ii] = \epsilon^{\dot\alpha\dot\beta}\ i]_{\dot\alpha} i]_{\dot\beta} = 0$$

Little group covariance

Little group transformations leave p_i invariant

Little group is $SO(2) \sim U(1)$ for massless p_i

Spinors $i], i\rangle$ pick up phases proportional $\pm 1/2$

The amplitude picks up a phase proportional to h_i

Massless three-points

fully determined by little-group covariance

$$\mathcal{M}(1^{h_1},2^{h_2},3^{h_3}) = g \begin{cases} [12]^{h_1+h_2-h_3} \quad [23]^{h_2+h_3-h_1} \quad [31]^{h_3+h_1-h_2} & \text{for } h_1+h_2+h_3>0 \\ \langle 12\rangle^{-h_1-h_2+h_3} \langle 23\rangle^{-h_2-h_3+h_1} \langle 31\rangle^{-h_3-h_1+h_2} & \text{for } h_1+h_2+h_3<0 \end{cases}$$

f^+f^+s $[12]$

v^+v^+s $[12]^2$

$f^+f^-v^+$ $[13]^2/[12]$

$v^+v^+v^-$ $[12]^3/[23][31]$

$t^+t^+t^-$ $\left([12]^3/[23][31]\right)^2$

Massless higher-points

Multiple independent structures for given helicities

satisfying · little group constraints
· momentum conservation
· Schouten identities e.g. $[12][34]-[13][24]+[14][23]=0$

Non-trivial Lorentz invariants

$$2\tilde{s}_{ij}=p_i\cdot p_j,\qquad \epsilon_{\mu\nu\rho\sigma}p_i^\mu p_j^\nu p_k^\rho p_l^\sigma$$

Construction

- *harmonics* (distinguishable particles) [Henning, Melia '19]
- *twistors* trivializing momentum conservation [Falkowski '19]
- systematic algorithm and explicit construction [GD, Machado '19]
[see also Accettulli Huber, De Angelis '21]

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e.g. minimal dimension of operators contributing to any helicity amplitude:

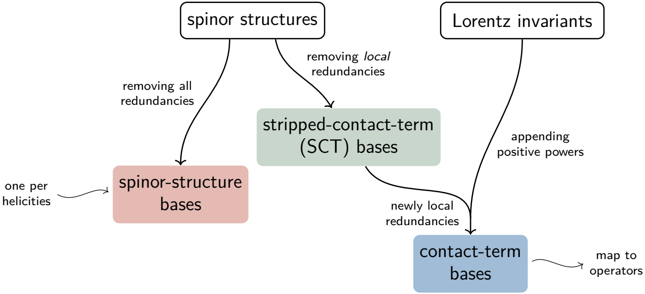
$$\dim\{\text{operator}\}\geq\ n\ -\sum_i \max(0,\text{ceil}\{|h_i|-1\})$$
$$+\sum_i |h_i|\ +2\max\begin{bmatrix}\{\sum_{h_i>0}2h_i\}\mod 2\\ 2\max\limits_{h_i>0}\{|h_i|\}-\sum\limits_{h_i>0}|h_i|\\ 2\max\limits_{h_i<0}\{|h_i|\}-\sum\limits_{h_i<0}|h_i|\end{bmatrix}$$

[GD, Machado '19]

Stripped contact terms (SCTs)

Strip out Lorentz invariants from spinors

$$\text{e.g. } [12][34] \times (\tilde{s}_{12} - \tilde{s}_{13})$$


spinor structures

Lorentz invariants

removing all redundancies

removing *local* redundancies

stripped-contact-term (SCT) bases

appending positive powers

one per helicities

spinor-structure bases

newly local redundancies

contact-term bases

map to operators

Exclude non-local relation in SCT basis reduction

- ✗ $[12][34] = -[13][24]\ \tilde{s}_{12}/\tilde{s}_{13}$
- ✓ $[12][34]\ \tilde{s}_{13} = -[13][24]\ \tilde{s}_{12}$

Stripped contact terms (SCTs)

Strip out

one per
helicities

Exclude r

- ✗ [12][3
- ✓ [12][3

e.g. GR-SMEFT [GD, Machado '19]

mult.	min. dim.	helicity conf.	spinor structures	SM gauge	spin stat.	Hilbert series
3-pt	dim-5	t^+t^+s	$[12]^4$	✗		
	dim-6	$t^+t^+t^+$	$[12]^2[13]^2[23]^2$			C_R^3
		$t^+t^+v^+$	$[12]^3[23][13]$		✗	
		$t^+v^+v^+$	$[12]^2[13]^2$			$(B_R^2, W_R^2, G_R^2)C_R$
4-pt	dim-6	$\underline{t}^+\underline{t}^+ss$	$[12]^4$; $[12]^4 s_{12}$			$HC_R^2H^\dagger$, $HD^2H^\dagger C_R^2$
	dim-7	$t^+t^+t^+s$	$[12]^2[13]^2[23]^2$	✗		
		$t^+t^+v^+s$	$[12]^3[13][23]$	✗		
		$\underline{t}^+\underline{t}^+f^+f^+$	$[12]^4[34]$	✗		
		$t^+t^+f^-f^-$	$[12]^4\langle 34\rangle$	✗		
		$t^+v^+v^+s$	$[12]^2[13]^2$	✗		
		$t^+v^+f^+f^+$	$[12]^2[13][14]$	✗		
	dim-8	$\underline{t}^+\underline{t}^+\underline{t}^+\underline{t}^+$	$[12]^4[34]^4+[13]^4[24]^4+[14]^4[23]^4$			C_R^4
		$t^+t^+t^+v^+$	$[12]^3[13][23][34]^2$, $[12][13]^3[23][24]^2$, $[12][13][23]^3[14]^2$		✗	
		$t^+t^+t^-t^-$	$[12]^4\langle 34\rangle^4$			$C_R^2C_L^2$
		$\underline{t}^+\underline{t}^+v^+v^+$	$[12]^4[34]^2$, $[12]^2[13][14][24][23]$			$2(B_R^2, W_R^2, G_R^2)C_R^2$
		$t^+t^+v^-v^-$	$[12]^4\langle 34\rangle^2$			$(B_L^2, W_L^2, G_L^2)C_R^2$
		$t^+t^+f^+f^-$	$[12]^4[324\rangle$		✗	
		$t^+v^+\underline{v}^+\underline{v}^+$	$[12][13][14]([13][24]+[14][23])$			$(W_R^2, G_R^2)B_RC_R$
		$t^+v^+f^+f^-$	$[12]^2[13][124\rangle$			$(QQ^\dagger, uu^\dagger, dd^\dagger, LL^\dagger, ee^\dagger)DB_RC_R$, $(QQ^\dagger, LL^\dagger)DW_RC_R$, $(QQ^\dagger, uu^\dagger, dd^\dagger)DG_RC_R$
		t^+v^+ss	$[12]^2[1231]$			$(B_R, W_R)HH^\dagger D^2C_R$
		$t^+f^+f^+s$	$[12][13][1231]$			$(Q^\dagger u^\dagger H^\dagger, Q^\dagger d^\dagger H, L^\dagger e^\dagger H)D^2C_R$
	...					
5-pt	dim-7	t^+t^+sss	$[12]^4$	✗		
	dim-8	$t^+t^+t^+ss$	$[12]^2[13]^2[23]^2$			$HH^\dagger C_R^3$
		$t^+t^+v^+ss$	$[12]^3[13][23]$		✗	
		$\underline{t}^+\underline{t}^+f^+f^+s$	$[12]^4[34]$			$(Q^\dagger u^\dagger H^\dagger, Q^\dagger d^\dagger H, L^\dagger e^\dagger H)C_R^2$
		$t^+t^+f^-f^-s$	$[12]^4\langle 34\rangle$			$(QuH, QdH^\dagger, LeH^\dagger)C_R^2$
		$t^+v^+v^+ss$	$[12]^2[13]^2$			$(B_R^2, B_RW_R, W_R^2, G_R^2)HH^\dagger C_R$
		$t^+v^+f^+f^+s$	$[12]^2[13][14]$			$(Q^\dagger u^\dagger H^\dagger, Q^\dagger d^\dagger H, L^\dagger e^\dagger H)(B_R, W_R)C_R$ $(Q^\dagger u^\dagger H^\dagger, Q^\dagger d^\dagger H)G_RC_R$
		$t^+f^+f^+f^+f^+$	$[12][13][14][15]$			$Q^\dagger Q^\dagger Q^\dagger L^\dagger C_R, d^\dagger e^\dagger u^{\dagger 2}C_R$, $d^\dagger Q^{\dagger 2}u^\dagger C_R, e^\dagger L^\dagger Q^\dagger u^\dagger C_R$
	...					
6-pt	dim-8	t^+t^+ssss	$[12]^4$			$H^2H^{\dagger 2}C_R^2$

Massless operator bases

adding gauge structures, treating identical particles

- $h/Z+$ gluons [Shadmi, Weiss '18]
- SMEFT at dim-6 [Ma, Shu, Xiao '19]
- SMEFT at dim-8 and 9 [Li, Ren, Shu, Xiao, Yu, Zheng '20]
  [see also Murphy '20]
  [Li, Ren, Xiao, Yu, Zheng '20]
- LEFT at dim-8 and 9 [Li Ren, Xiao, Yu, Zheng '20]
  [see also Murphy '20]
- GR-SMEFT at dim-8 [GD, Machado '19]
  [see also Ruhdorfer, Serra, Weiler '19]

b. Massive contact terms

Spin spinors

[Arkani-Hamed, Huang, Huang '17]

Two massless for one massive

$$p^i_\mu \sigma^\mu_{\alpha\dot\alpha} \;=\; k^i\rangle[k^i + q^i\rangle[q^i \;=\; i^J\rangle[i_J \qquad \text{with } \begin{array}{c} k_i^2 = 0 = q_i^2, \quad J=1,2 \\ 2k^i\cdot q^i = m_i^2 \end{array}$$

Little group is now $SO(3) \sim SU(2)$

Spin s from $2s$ symmetrized spin $1/2$

Bolded spinors with implicit $SU(2)$ index symmetrization

e.g. $\langle 1^I 3^J\rangle\langle 2^K 3^{J'}\rangle + (J \leftrightarrow J') \to \langle \mathbf{13}\rangle\langle \mathbf{23}\rangle$

ffs $[\mathbf{12}],\ \langle\mathbf{12}\rangle$

vvs $\langle\mathbf{12}\rangle^2,\ \langle\mathbf{12}\rangle[\mathbf{12}],\ [\mathbf{12}]^2$

ssv $[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \equiv [\mathbf{3}(p_\mathbf{1}-p_\mathbf{2})\mathbf{3}\rangle$

ffv $\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle,\ \langle\mathbf{13}\rangle[\mathbf{23}],\ [\mathbf{13}]\langle\mathbf{23}\rangle,\ [\mathbf{13}][\mathbf{23}]$

...

High-energy limit

- choice for the decomposition $p^\mu = (E, p\,\hat{n}) = k^\mu + q^\mu$:

$$k^\mu = \frac{E+p}{2}(1, +\hat{n}), \qquad q^\mu = \frac{E-p}{2}(1, -\hat{n})$$

$\rightarrow$ spin quantization axis $k^\mu - q^\mu \sim (1, \hat{n}) \rightsquigarrow$ helicity

High-energy limit

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$\rightarrow$ spin quantization axis $k^\mu - q^\mu \sim (1, \hat{n}) \rightsquigarrow$ helicity

$\rightarrow$ $k], k\rangle \sim \sqrt{E}$ and $q], q\rangle \sim m/\sqrt{E}$

- massless limit: *un-bolding* $+\mathcal{O}(m)$

e.g. $\langle \mathbf{13}\rangle\langle \mathbf{23}\rangle \rightarrow \langle 13\rangle\langle 23\rangle + \mathcal{O}(m)$

$$\langle \mathbf{13}\rangle[\mathbf{23}] \rightarrow \cancel{\langle 1 p_3 2]} + \mathcal{O}(m)$$

Massive three-points

Counting from angular momentum

number of irreps in the spin addition: [Costa, Penedones, Poland, Rychkov '11]

$$(2s_1+1)(2s_2+1)-p(p+1) \quad \text{with} \quad \begin{cases} p \equiv \max\{0, s_1+s_2-s_3\} \\ s_1 \leq s_2 \leq s_3 \end{cases}$$

Construction by correcting a massless-like ansatz

[GD, Kitahara, Machado, Shadmi, Weiss '20]

$$(\mathbf{12})^{s_1+s_2-\tilde{s}_3}\,(\mathbf{23})^{-s_1+s_2+\tilde{s}_3}\,(\mathbf{13})^{s_1-s_2+\tilde{s}_3}\,[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^{s_3-\tilde{s}_3}$$

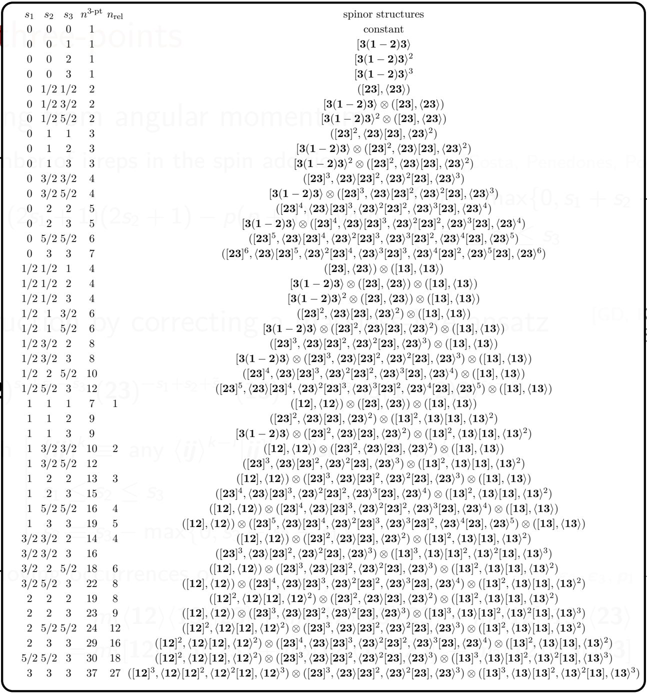
with

$$\left| \begin{array}{l} (\boldsymbol{ij})^k \equiv \text{ any } \langle \boldsymbol{ij}\rangle^{k-l}[\boldsymbol{ij}]^l \quad \text{for } l=0,...,k \\ s_1 \leq s_2 \leq s_3 \\ \tilde{s}_3 \equiv s_3 - \max\{0, s_3-s_2-s_1\} \end{array} \right.$$

removing occurrences of $\epsilon(\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \boldsymbol{\varepsilon}_3, \boldsymbol{p_1}+\boldsymbol{p_2}+\boldsymbol{p_3})$

$$\begin{aligned} & m_1\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}] + m_2\langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle + m_3[\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle \\ = {} & m_1[\mathbf{12}]\,[\mathbf{13}]\langle\mathbf{23}\rangle + m_2[\mathbf{12}]\langle\mathbf{13}\rangle[\mathbf{23}] + m_3\langle\mathbf{12}\rangle[\mathbf{13}]\,[\mathbf{23}] \end{aligned}$$

Massive three-points

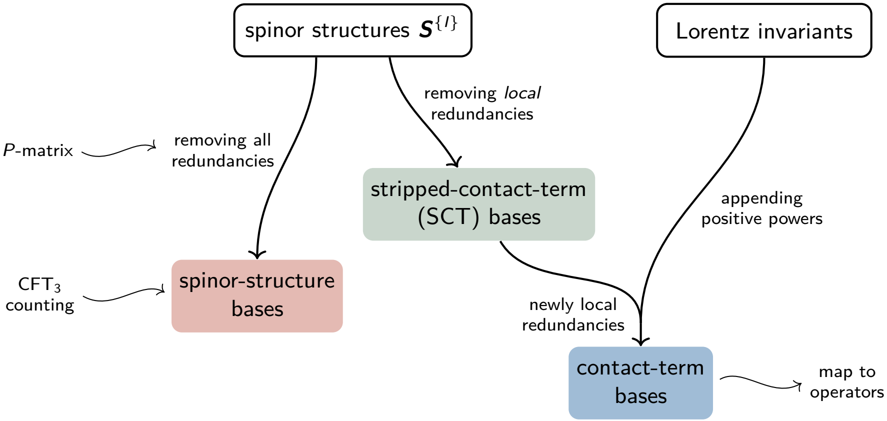


s_1	s_2	s_3	$n^{\text{3-pt}}$	n_{rel}	spinor structures
0	0	0	1		constant
0	0	1	1		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle$
0	0	2	1		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^2$
0	0	3	1		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^3$
0	1/2	1/2	2		$([\mathbf{23}], \langle\mathbf{23}\rangle)$
0	1/2	3/2	2		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \otimes ([\mathbf{23}], \langle\mathbf{23}\rangle)$
0	1/2	5/2	2		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^2 \otimes ([\mathbf{23}], \langle\mathbf{23}\rangle)$
0	1	1	3		$([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2)$
0	1	2	3		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2)$
0	1	3	3		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^2 \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2)$
0	3/2	3/2	4		$([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3)$
0	3/2	5/2	4		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3)$
0	2	2	5		$([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4)$
0	2	3	5		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \otimes ([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4)$
0	5/2	5/2	6		$([\mathbf{23}]^5, \langle\mathbf{23}\rangle[\mathbf{23}]^4, \langle\mathbf{23}\rangle^2[\mathbf{23}]^3, \langle\mathbf{23}\rangle^3[\mathbf{23}]^2, \langle\mathbf{23}\rangle^4[\mathbf{23}], \langle\mathbf{23}\rangle^5)$
0	3	3	7		$([\mathbf{23}]^6, \langle\mathbf{23}\rangle[\mathbf{23}]^5, \langle\mathbf{23}\rangle^2[\mathbf{23}]^4, \langle\mathbf{23}\rangle^3[\mathbf{23}]^3, \langle\mathbf{23}\rangle^4[\mathbf{23}]^2, \langle\mathbf{23}\rangle^5[\mathbf{23}], \langle\mathbf{23}\rangle^6)$
1/2	1/2	1	4		$([\mathbf{23}], \langle\mathbf{23}\rangle) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	1/2	2	4		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \otimes ([\mathbf{23}], \langle\mathbf{23}\rangle) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	1/2	3	4		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^2 \otimes ([\mathbf{23}], \langle\mathbf{23}\rangle) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	1	3/2	6		$([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	1	5/2	6		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	3/2	2	8		$([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	3/2	3	8		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	2	5/2	10		$([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	5/2	3	12		$([\mathbf{23}]^5, \langle\mathbf{23}\rangle[\mathbf{23}]^4, \langle\mathbf{23}\rangle^2[\mathbf{23}]^3, \langle\mathbf{23}\rangle^3[\mathbf{23}]^2, \langle\mathbf{23}\rangle^4[\mathbf{23}], \langle\mathbf{23}\rangle^5) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1	1	1	7	1	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}], \langle\mathbf{23}\rangle) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1	1	2	9		$([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
1	1	3	9		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
1	3/2	3/2	10	2	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1	3/2	5/2	12		$([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
1	2	2	13	3	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1	2	3	15		$([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
1	5/2	5/2	16	4	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1	3	3	19	5	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}]^5, \langle\mathbf{23}\rangle[\mathbf{23}]^4, \langle\mathbf{23}\rangle^2[\mathbf{23}]^3, \langle\mathbf{23}\rangle^3[\mathbf{23}]^2, \langle\mathbf{23}\rangle^4[\mathbf{23}], \langle\mathbf{23}\rangle^5) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
3/2	3/2	2	14	4	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
3/2	3/2	3	16		$([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}]^3, \langle\mathbf{13}\rangle[\mathbf{13}]^2, \langle\mathbf{13}\rangle^2[\mathbf{13}], \langle\mathbf{13}\rangle^3)$
3/2	2	5/2	18	6	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
3/2	5/2	3	22	8	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
2	2	2	19	8	$([\mathbf{12}]^2, \langle\mathbf{12}\rangle[\mathbf{12}], \langle\mathbf{12}\rangle^2) \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
2	2	3	23	9	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}]^3, \langle\mathbf{13}\rangle[\mathbf{13}]^2, \langle\mathbf{13}\rangle^2[\mathbf{13}], \langle\mathbf{13}\rangle^3)$
2	5/2	5/2	24	12	$([\mathbf{12}]^2, \langle\mathbf{12}\rangle[\mathbf{12}], \langle\mathbf{12}\rangle^2) \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
2	3	3	29	16	$([\mathbf{12}]^2, \langle\mathbf{12}\rangle[\mathbf{12}], \langle\mathbf{12}\rangle^2) \otimes ([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
5/2	5/2	3	30	18	$([\mathbf{12}]^2, \langle\mathbf{12}\rangle[\mathbf{12}], \langle\mathbf{12}\rangle^2) \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}]^3, \langle\mathbf{13}\rangle[\mathbf{13}]^2, \langle\mathbf{13}\rangle^2[\mathbf{13}], \langle\mathbf{13}\rangle^3)$
3	3	3	37	27	$([\mathbf{12}]^3, \langle\mathbf{12}\rangle[\mathbf{12}]^2, \langle\mathbf{12}\rangle^2[\mathbf{12}], \langle\mathbf{12}\rangle^3) \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}]^3, \langle\mathbf{13}\rangle[\mathbf{13}]^2, \langle\mathbf{13}\rangle^2[\mathbf{13}], \langle\mathbf{13}\rangle^3)$

$\tilde{s}_{ij} \equiv 2p_i \cdot p_j$

Massive higher-points

e.g. $W^+W^+W^-W^-$: $$\frac{[\mathbf{13}][\mathbf{24}]\langle\mathbf{13}\rangle\langle\mathbf{24}\rangle - [\mathbf{14}][\mathbf{23}]\langle\mathbf{14}\rangle\langle\mathbf{23}\rangle}{m_1 m_2 m_3 m_4} \quad (\tilde{s}_{13} - \tilde{s}_{14} - \tilde{s}_{23} + \tilde{s}_{24})$$



spinor structures $\boldsymbol{S}^{\{I\}}$

Lorentz invariants

removing *local* redundancies

P-matrix

removing all redundancies

stripped-contact-term (SCT) bases

appending positive powers

CFT_3 counting

spinor-structure bases

newly local redundancies

contact-term bases

map to operators

[GD, Kitahara, Machado, Shadmi, Weiss ’20]

An aside: massive spinor structures

Counting from 3D conformal correlators

[Henning, Lu, Melia, Murayama '17]
[Shomerus, Sobko, Isachenkov '16]
[Kravchuk, Dimmons-Duffin '16]

$$\prod_i (2s_i + 1)$$

Independence test with P-matrix determinant

[Bonifacio, Hinterbichler '18]

inner product between spinor structures of identical spin content

$$P_{m,n} \equiv \sum_{\{I\},\{J\}} \boldsymbol{S}_m^{\{I\}*} \boldsymbol{S}_n^{\{J\}} \; \delta_{\{I\},\{J\}}$$

Construction by bolding massless amps

[GD, Kitahara, Machado,
Shadmi Weiss '20]

- Pick spinor structures whose leading high-energy limits contribute to different helicity amplitudes.
- Their P-matrix is diagonal in the massless limit as they do not interfere. So they are independent.
- There are $\prod_i (2s_i + 1)$ of them. So they form a basis.

Massive higher-points

[GD, Kitahara, Machado, Shadmi, Weiss '20]

New massive redundancies in SCT basis reduction

e.g. in *ffvs*: $$[\mathbf{12}]\langle\mathbf{3123}\rangle = ([\mathbf{12}][\mathbf{313}\rangle\tilde{s}_{23} - [\mathbf{12}][\mathbf{323}\rangle\tilde{s}_{13})/m_3 \\ - \tilde{s}_{12}[\mathbf{13}][\mathbf{23}] - m_1[\mathbf{321}\rangle[\mathbf{23}] - m_2[\mathbf{312}\rangle[\mathbf{13}]$$

Clear massless origins but non-trivial *mass-completions*

Massive higher-points

[GD, Kitahara, Machado, Shadmi, Weiss '20]

New massive redundancies in SCT basis reduction

e.g. in *ffvs*: $$[\mathbf{12}]\langle\mathbf{3123}\rangle = ([\mathbf{12}][\mathbf{313}\rangle\tilde{s}_{23} - [\mathbf{12}][\mathbf{323}\rangle\tilde{s}_{13})/m_3 \\ - \tilde{s}_{12}[\mathbf{13}][\mathbf{23}] - m_1[\mathbf{321}\rangle[\mathbf{23}] - m_2[\mathbf{312}\rangle[\mathbf{13}]$$

Clear massless origins but non-trivial *mass-completions*

e.g. in *vvff*: $$\begin{aligned}&[\mathbf{13}]\langle\mathbf{24}\rangle\langle\mathbf{231}] + [\mathbf{23}]\langle\mathbf{14}\rangle\langle\mathbf{132}] = \\ &- ([\mathbf{13}]\langle\mathbf{24}\rangle\langle\mathbf{132}](m_1^2 + \tilde{s}_{14}) + [\mathbf{23}]\langle\mathbf{14}\rangle\langle\mathbf{231}](m_1^2 + \tilde{s}_{13}))/m_1 m_2 \\ &+ [\mathbf{12}]([\mathbf{13}]\langle\mathbf{24}\rangle\tilde{s}_{23}/m_2 - [\mathbf{23}]\langle\mathbf{14}\rangle\tilde{s}_{13}/m_1) \\ &+ [\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{24}\rangle(m_1^2 + \tilde{s}_{14})m_3/m_1 m_2 \\ &- [\mathbf{12}]\langle\mathbf{24}\rangle\langle\mathbf{321}]m_3/m_2 + [\mathbf{34}]\langle\mathbf{12}\rangle\langle\mathbf{132}]m_4/m_2 \\ &- \langle\mathbf{12}\rangle([\mathbf{23}]\langle\mathbf{14}\rangle m_3 + [\mathbf{24}]\langle\mathbf{13}\rangle m_4)m_3/m_2 \\ &- [\mathbf{14}][\mathbf{23}]\langle\mathbf{12}\rangle\tilde{s}_{13} m_4/m_1 m_2 \,.\end{aligned}$$

Explicit construction for 4-points and spins$\leq$1

Massive higher-points

[GD, Kitahara, Machado, Shadmi, Weiss '20]

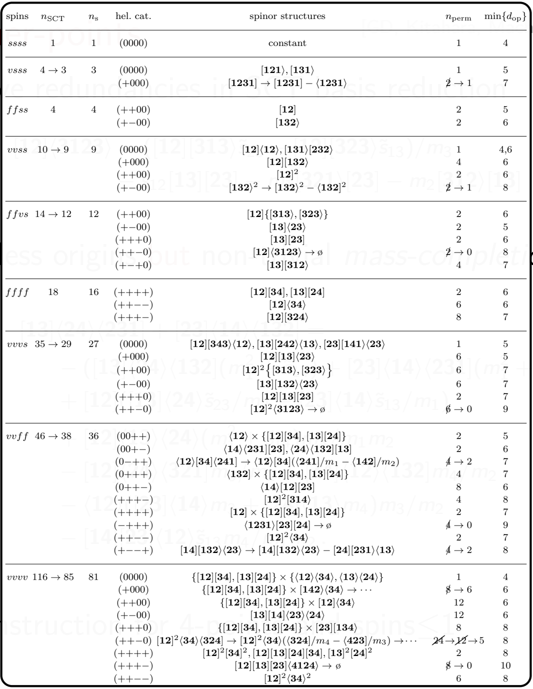
New massive redundancies in SCT basis reduction

e.g. in *ffvs*:

Clear massless origins but non-trivial *mass-completions*

e.g. in *vvff*:

Explicit construction for 4-points and spins≤1



spins	n_{SCT}	n_s	hel. cat.	spinor structures	n_{perm}	$\min\{d_{\text{op}}\}$
$ssss$	1	1	(0000)	constant	1	4
$vsss$	$4\to3$	3	(0000)	$[121\rangle, [131\rangle$	1	5
			(+000)	$[1231] \to [1231] - \langle 1231\rangle$	$\cancel{2}\to1$	7
$ffss$	4	4	(++00)	$[12]$	2	5
			(+−00)	$[132\rangle$	2	6
$vvss$	$10\to9$	9	(0000)	$[12]\langle12\rangle, [131\rangle[232\rangle$	1	4,6
			(+000)	$[12][132\rangle$	4	6
			(++00)	$[12]^2$	2	6
			(+−00)	$[132\rangle^2 \to [132\rangle^2 - \langle132]^2$	$\cancel{2}\to1$	8
$ffvs$	$14\to12$	12	(++00)	$[12]\{[313\rangle, [323\rangle\}$	2	6
			(+−00)	$[13]\langle23\rangle$	2	5
			(+++0)	$[13][23]$	2	6
			(++−0)	$[12]\langle3123\rangle \to \varnothing$	$\cancel{2}\to0$	8
			(+−+0)	$[13][312\rangle$	4	7
$ffff$	18	16	(++++)	$[12][34], [13][24]$	2	6
			(++−−)	$[12]\langle34\rangle$	6	6
			(+++−)	$[12][324\rangle$	8	7
$vvvs$	$35\to29$	27	(0000)	$[12][343\rangle\langle12\rangle, [13][242\rangle\langle13\rangle, [23][141\rangle\langle23\rangle$	1	5
			(+000)	$[12][13\rangle\langle23\rangle$	6	5
			(++00)	$[12]^2\left\{[313\rangle, [323\rangle\right\}$	6	7
			(+−00)	$[13][132\rangle\langle23\rangle$	6	7
			(+++0)	$[12][13][23]$	2	7
			(++−0)	$[12]^2\langle3123\rangle \to \varnothing$	$\cancel{6}\to0$	9
$vvff$	$46\to38$	36	(00++)	$\langle12\rangle \times \{[12][34], [13][24]\}$	2	5
			(00+−)	$\langle14\rangle\langle231][23], \langle24\rangle\langle132][13]$	2	6
			(0−++)	$\langle12\rangle[34]\langle241] \to \langle12\rangle[34](\langle241]/m_1 - \langle142]/m_2)$	$\cancel{4}\to2$	7
			(0+++)	$\langle132] \times \{[12][34], [13][24]\}$	4	7
			(0++−)	$\langle14\rangle[12][23]$	8	6
			(+++−)	$[12]^2[314\rangle$	4	8
			(++++)	$[12] \times \{[12][34], [13][24]\}$	2	7
			(−+++)	$\langle1231\rangle[23][24] \to \varnothing$	$\cancel{4}\to0$	9
			(++−−)	$[12]^2\langle34\rangle$	2	7
			(+−−+)	$[14][132\rangle\langle23\rangle \to [14][132\rangle\langle23\rangle - [24][231\rangle\langle13\rangle$	$\cancel{4}\to2$	8
$vvvv$	$116\to85$	81	(0000)	$\{[12][34], [13][24]\} \times \{\langle12\rangle\langle34\rangle, \langle13\rangle\langle24\rangle\}$	1	4
			(+000)	$\{[12][34], [13][24]\} \times [142\rangle\langle34\rangle \to \cdots$	$\cancel{8}\to6$	6
			(++00)	$\{[12][34], [13][24]\} \times [12]\langle34\rangle$	12	6
			(+−00)	$[13][14]\langle23\rangle\langle24\rangle$	12	6
			(+++0)	$\{[12][34], [13][24]\} \times [23][134\rangle$	8	8
			(++−0)	$[12]^2\langle34\rangle\langle324] \to [12]^2\langle34\rangle(\langle324]/m_4 - \langle423]/m_3) \to \cdots$	$\cancel{24}\to\cancel{12}\to5$	8
			(++++)	$[12]^2[34]^2, [12][13][24][34], [13]^2[24]^2$	2	8
			(+++−)	$[12][13][23]\langle4124\rangle \to \varnothing$	$\cancel{8}\to0$	10
			(++−−)	$[12]^2\langle34\rangle^2$	6	8

Massive electroweak EFTs

Assumptions

Poincaré, locality, unitarity

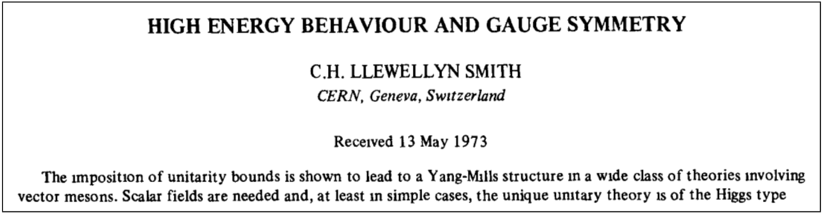
electroweak spectrum: $\psi, \psi', \gamma, Z, W^\pm, h$
(all massive but γ)

global electric charge & fermion number
$|Q_\psi - Q_{\psi'}| = |Q_W|$

Recovering electroweak symmetry

Given a particle content,
from perturbative unitarity up to $\bar{\Lambda} \gg m$!

[Llewelly Smith '73]
[Joglekar '73]
[Conwall et al. '73, '74]

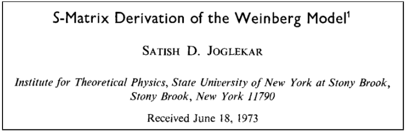


HIGH ENERGY BEHAVIOUR AND GAUGE SYMMETRY

C.H. LLEWELLYN SMITH
CERN, Geneva, Switzerland

Received 13 May 1973

The imposition of unitarity bounds is shown to lead to a Yang-Mills structure in a wide class of theories involving vector mesons. Scalar fields are needed and, at least in simple cases, the unique unitary theory is of the Higgs type



S-Matrix Derivation of the Weinberg Model[1]

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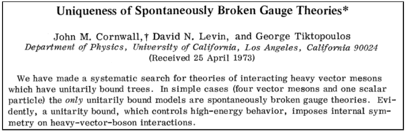
Received June 18, 1973



Uniqueness of Spontaneously Broken Gauge Theories*

John M. Cornwall,† David N. Levin, and George Tiktopoulos
Department of Physics, University of California, Los Angeles, California 90024
(Received 25 April 1973)

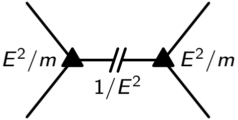
We have made a systematic search for theories of interacting heavy vector mesons which have unitarily bound trees. In simple cases (four vector mesons and one scalar particle) the *only* unitarily bound models are spontaneously broken gauge theories. Evidently, a unitarity bound, which controls high-energy behavior, imposes internal symmetry on heavy-vector-boson interactions.

a. Massive EW three-points

Perturbative unitarity at three-point?

Rule of thumb:

Three-points growing as E^2/m or faster (for complex momenta) lead to unacceptable E^2/m^2 four-point growths.



$\sim E^2/m^2$

not considered here

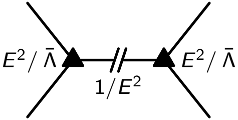
(the dependence of the spinor structure coefficient on the propagator virtuality could cancel the E growth)

$\rightarrow$ E^2 three-points only arise · at the loop level

Perturbative unitarity at three-point?

Rule of thumb:

Three-points growing as E^2/m or faster (for complex momenta) lead to unacceptable E^2/m^2 four-point growths.



$\sim E^2/\bar{\Lambda}^2$

not considered here

(the dependence of the spinor structure coefficient on the propagator virtuality could cancel the E growth)

→ E^2 three-points only arise
- at the loop level
- at the $1/\bar{\Lambda}$ level

$\psi^c\psi Z$

$$\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle,\ \langle\mathbf{13}\rangle[\mathbf{23}],\ [\mathbf{13}]\langle\mathbf{23}\rangle,\ [\mathbf{13}][\mathbf{23}]$$

four terms expected from ang. mom.: $2 \otimes 2 \otimes 3 = 1 \oplus 3 \oplus 3 \oplus 5$

$\psi^c\psi Z$

$\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle$, $\langle\mathbf{13}\rangle[\mathbf{23}]$, $[\mathbf{13}]\langle\mathbf{23}\rangle$, $[\mathbf{13}][\mathbf{23}]$

high energy

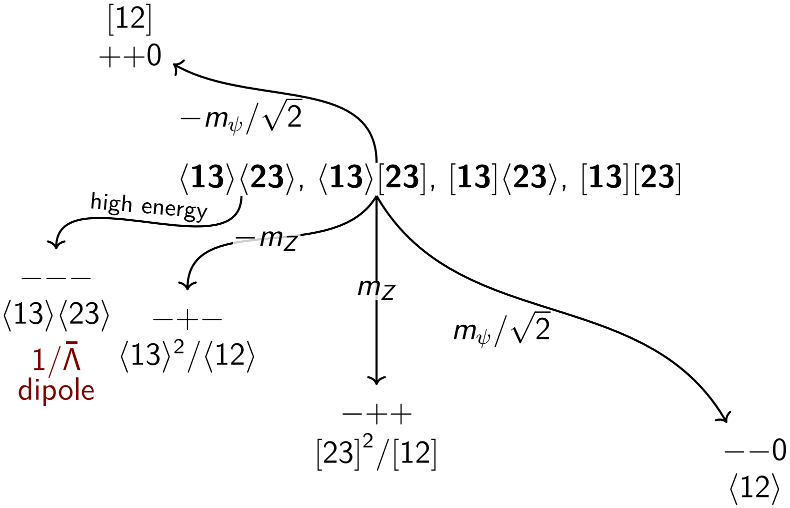
$---$

$\langle 13\rangle\langle 23\rangle$

$1/\bar{\Lambda}$

dipole

$\psi^c \psi Z$



$[12]$
$++0$

$-m_\psi/\sqrt{2}$

$\langle \mathbf{13} \rangle \langle \mathbf{23} \rangle$, $\langle \mathbf{13} \rangle [\mathbf{23}]$, $[\mathbf{13}] \langle \mathbf{23} \rangle$, $[\mathbf{13}][\mathbf{23}]$

high energy

$-m_Z$

m_Z

$m_\psi/\sqrt{2}$

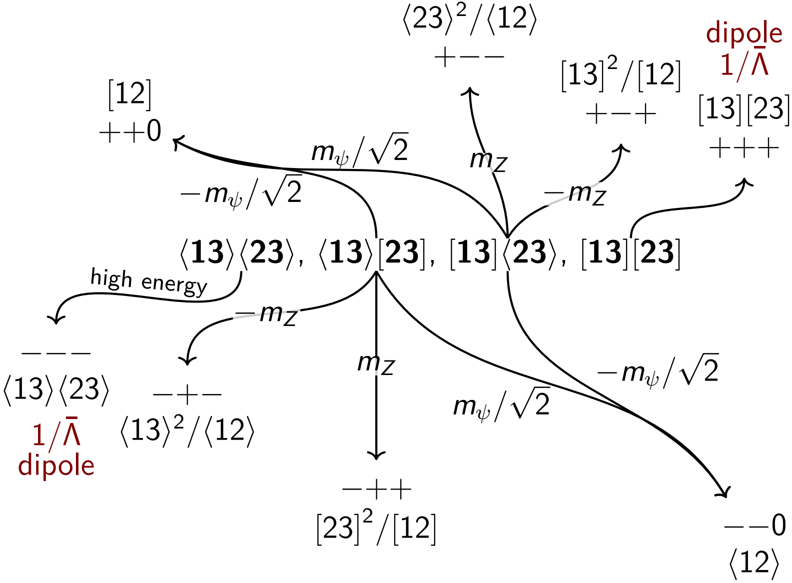
$---$
$\langle 13 \rangle \langle 23 \rangle$
$1/\bar{\Lambda}$
dipole

$-+-$
$\langle 13 \rangle^2 / \langle 12 \rangle$

$-++$
$[23]^2/[12]$

$--0$
$\langle 12 \rangle$

$\psi^c\psi Z$



$\langle 23\rangle^2/\langle 12\rangle$

$+--$

$[13]^2/[12]$

$+-+$

dipole

$1/\bar{\Lambda}$

$[13][23]$

$+++$

$[12]$

$++0$

$m_\psi/\sqrt{2}$

m_Z

$-m_\psi/\sqrt{2}$

$-m_Z$

$\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle$, $\langle\mathbf{13}\rangle[\mathbf{23}]$, $[\mathbf{13}]\langle\mathbf{23}\rangle$, $[\mathbf{13}][\mathbf{23}]$

high energy

$-m_Z$

m_Z

$-m_\psi/\sqrt{2}$

$m_\psi/\sqrt{2}$

$---$

$\langle 13\rangle\langle 23\rangle$

$1/\bar{\Lambda}$

dipole

$-+-$

$\langle 13\rangle^2/\langle 12\rangle$

$-++$

$[23]^2/[12]$

$--0$

$\langle 12\rangle$

$\psi^c \psi Z$

$$\left.\frac{\text{longitudinal}}{\text{transverse}}\right|_{\text{high } E} \sim \quad \frac{g^L - g^R}{g^{L,R}} \; \frac{m_\psi}{\sqrt{2} m_Z}$$

$\rightarrow \sim y_\psi$

$\rightarrow$ pseudo-scalar

$\psi^c \psi Z$

$$\left.\frac{\text{longitudinal}}{\text{transverse}}\right|_{\text{high } E} \sim \quad \frac{g^L - g^R}{g^{L,R}} \; \frac{m_\psi}{\sqrt{2} m_Z}$$

$\rightarrow \sim y_\psi$

$\rightarrow$ pseudo-scalar

reproduces expectations
from the Higgs mechanism
and the Goldstone equivalence theorem

more in the four-point discussion

W^+W^-Z

- 8 combinations of $\begin{matrix}\langle\mathbf{12}\rangle\\ [\mathbf{12}]\end{matrix} \otimes \begin{matrix}\langle\mathbf{23}\rangle\\ [\mathbf{23}]\end{matrix} \otimes \begin{matrix}\langle\mathbf{31}\rangle\\ [\mathbf{31}]\end{matrix}$

- one non-trivial relation between them:

$$\begin{aligned} & m_1\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}] + m_2\langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle + m_3[\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle \\ = & m_1[\mathbf{12}]\,[\mathbf{13}]\langle\mathbf{23}\rangle + m_2[\mathbf{12}]\langle\mathbf{13}\rangle[\mathbf{23}] + m_3\langle\mathbf{12}\rangle[\mathbf{13}]\,[\mathbf{23}] \end{aligned}$$

7 combinations expected from angular momentum
$3 \otimes 3 \otimes 3 = 1 \oplus 3 \oplus 3 \oplus 3 \oplus 5 \oplus 5 \oplus 7$

- combinations growing like E^3 and E^2
can only arise at the non-renormalizable (tree) level

W^+W^-Z

C is $\mathbf{1} \leftrightarrow \mathbf{2}$

P is $\cdot] \leftrightarrow \cdot\rangle$

$$
\begin{aligned}
&\mathcal{M}(\mathbf{1}_W, \mathbf{2}_W, \mathbf{3}_Z) = \\
&\quad + \langle\mathbf{12}\rangle\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle\; c_{WWZ}^{LLL}/\bar{\Lambda}^2 \\
&\quad + \langle\mathbf{12}\rangle(\langle\mathbf{13}\rangle[\mathbf{23}] - [\mathbf{13}]\langle\mathbf{23}\rangle)\; c_{WWZ}^{[L0]0}/m_Z\bar{\Lambda} \\
&\quad + \langle\mathbf{12}\rangle(\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{13}]\langle\mathbf{23}\rangle)\; c_{WWZ}^{\{L0\}0}/m_Z\bar{\Lambda} \\
&\quad + \big\{[\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle\; m_Z/m_W + \langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle + \langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}] \\
&\quad + \langle\mathbf{12}\rangle\,[\mathbf{13}]\,[\mathbf{23}]\; m_Z/m_W + [\mathbf{12}]\,\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{12}]\,[\mathbf{13}]\,\langle\mathbf{23}\rangle\big\}\; c_{WWZ}/m_Z m_W \\
&\quad + [\mathbf{12}](\langle\mathbf{13}\rangle[\mathbf{23}] - [\mathbf{13}]\langle\mathbf{23}\rangle)\; c_{WWZ}^{[R0]0}/m_Z\bar{\Lambda} \\
&\quad + [\mathbf{12}](\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{13}]\langle\mathbf{23}\rangle)\; c_{WWZ}^{\{R0\}0}/m_Z\bar{\Lambda} \\
&\quad + [\mathbf{12}][\mathbf{13}][\mathbf{23}]\; c_{WWZ}^{RRR}/\bar{\Lambda}^2\,,
\end{aligned}
$$

- one single (rather non-trivial) renormalizable structure!

W^+W^-Z

C is $\mathbf{1} \leftrightarrow \mathbf{2}$

P is $\cdot] \leftrightarrow \cdot\rangle$

$$
\begin{aligned}
\mathcal{M}(\mathbf{1}_W, \mathbf{2}_W, \mathbf{3}_Z) = \\
&+ \langle\mathbf{12}\rangle\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle\ c_{WWZ}^{LLL}/\bar{\Lambda}^2 \\
&+ \langle\mathbf{12}\rangle(\langle\mathbf{13}\rangle[\mathbf{23}] - [\mathbf{13}]\langle\mathbf{23}\rangle)\ c_{WWZ}^{[L0]0}/m_Z\bar{\Lambda} \\
&+ \langle\mathbf{12}\rangle(\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{13}]\langle\mathbf{23}\rangle)\ c_{WWZ}^{\{L0\}0}/m_Z\bar{\Lambda} \\
&+ \big\{[\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle\ m_Z/m_W + \langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle + \langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}] \\
&+ \langle\mathbf{12}\rangle\,[\mathbf{13}]\,[\mathbf{23}]\ m_Z/m_W + [\mathbf{12}]\,\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{12}]\,[\mathbf{13}]\,\langle\mathbf{23}\rangle\big\}\ c_{WWZ}/m_Z m_W \\
&+ [\mathbf{12}](\langle\mathbf{13}\rangle[\mathbf{23}] - [\mathbf{13}]\langle\mathbf{23}\rangle)\ c_{WWZ}^{[R0]0}/m_Z\bar{\Lambda} \\
&+ [\mathbf{12}](\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{13}]\langle\mathbf{23}\rangle)\ c_{WWZ}^{\{R0\}0}/m_Z\bar{\Lambda} \\
&+ [\mathbf{12}][\mathbf{13}][\mathbf{23}]\ c_{WWZ}^{RRR}/\bar{\Lambda}^2\,,
\end{aligned}
$$

- one single (rather non-trivial) renormalizable structure!
- for identical vectors ($m_Z/m_W \to 1$)
  - no fully symmetric combination → ZZZ vanishes
  - only fully antisymmetric combinations → $W^aW^bW^c$ requires ϵ_{abc}

All three-points

$$\psi^c \psi Z$$

$$\psi^c \psi \gamma$$

$$\psi^c \psi' W$$

$$\psi^c \psi h$$

$$ZZh,\ WWh$$

$$\gamma\gamma h,\ \gamma Zh$$

$$hhZ,\ hh\gamma$$

$$hhh$$

$$WWZ$$

$$WW\gamma$$

$$ZZZ,\ ZZ\gamma,\ Z\gamma\gamma,\ \gamma\gamma\gamma$$

All three-points

$\psi^c \psi Z$

$\psi^c \psi \gamma$

$\psi^c \psi' W$

$\psi^c \psi h$

ZZh, WWh

$\gamma\gamma h$, $\gamma Z h$

hhZ, $hh\gamma$

hhh

WWZ

$WW\gamma$

ZZZ, $ZZ\gamma$, $Z\gamma\gamma$, $\gamma\gamma\gamma$

some vanish identically
some only arise at the non-renormalizable (tree) level
some can be obtained from the massless limit of others

[see also renormalizable discussion in Christensen et al. '18]

Matching to SMEFT

Extending results from [Aoude, Machado '19]
Warsaw basis conventions from [Dedes et al. '17]

Three-point amplitude coefficients capture all orders in v/Λ.

e.g. $\mathcal{M}(\mathbf{1}_{\psi^c},\mathbf{2}_{\psi},\mathbf{3}_Z)=\frac{c^{LLL}_{\psi^c\psi Z}}{\bar\Lambda}\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle+\frac{c^{LR0}_{\psi^c\psi Z}}{m_Z}\langle\mathbf{13}\rangle[\mathbf{23}]+\frac{c^{RL0}_{\psi^c\psi Z}}{m_Z}[\mathbf{13}]\langle\mathbf{23}\rangle+\frac{c^{RRR}_{\psi^c\psi Z}}{\bar\Lambda}[\mathbf{13}][\mathbf{23}]$

at dim-6:

$$c^{LR0}_{\psi^c\psi Z}=-\sqrt{2}Q_\psi\frac{\bar g'^2}{\sqrt{\bar g^2+\bar g'^2}}+\frac{v^2}{\Lambda^2}\left[-\sqrt{2}\frac{\bar g^3\bar g'}{(\bar g^2+\bar g'^2)^{3/2}}Q_\psi C_{\varphi WB}-\frac{1}{\sqrt{2}}\sqrt{\bar g^2+\bar g'^2}C_{\varphi\psi_R}\right]$$

$$\begin{aligned}c^{RL0}_{\psi^c\psi Z}=&\sqrt{2}I_\psi\frac{\bar g^2}{\sqrt{\bar g^2+\bar g'^2}}-\sqrt{2}Y_\psi\frac{\bar g'^2}{\sqrt{\bar g^2+\bar g'^2}}\\&+\frac{v^2}{\Lambda^2}\left[\sqrt{2}\frac{\bar g\bar g'}{(\bar g^2+\bar g'^2)^{3/2}}\left(-Y_\psi\bar g^2+I_\psi\bar g'^2\right)C_{\varphi WB}\right.\\&\left.-\frac{1}{\sqrt{2}}\sqrt{\bar g^2+\bar g'^2}\left(C^1_{\varphi\psi_L}-2I_\psi C^3_{\varphi\psi_L}\right)\right]\end{aligned}$$

$$\frac{c^{RRR}_{\psi^c\psi Z}}{\bar\Lambda}=\frac{v}{\Lambda^2}\left(-4I_\psi\frac{\bar g}{\sqrt{\bar g^2+\bar g'^2}}C_{\psi W}+2\frac{\bar g'}{\sqrt{\bar g^2+\bar g'^2}}C_{\psi B}\right)$$

$$c^{LLL}_{\psi^c\psi Z}=\left(c^{RRR}_{\psi^c\psi Z}\right)^*$$

$I_{u,d,e,\nu}\equiv {}^1/_2,\ -{}^1/_2,\ -{}^1/_2,\ {}^1/_2$
$Y_{u,d,e,\nu}\equiv {}^1/_6,\ {}^1/_6,\ -{}^1/_2,\ -{}^1/_2$

b. Full four-point example: $\psi^c \psi Z h$

contact + factorizable terms

Contact terms

captures all EFT orders
both in HEFT and SMEFT

- Twelve independent SCTs:

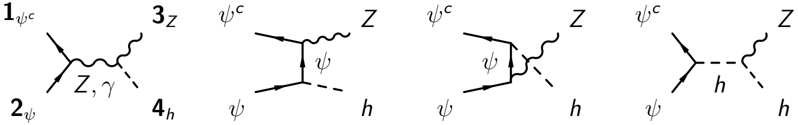
$$
\begin{aligned}
\mathcal{M}^{\text{nf}}(\mathbf{1}_{\psi^c},\mathbf{2}_{\psi},\mathbf{3}_Z,\mathbf{4}_h) = {} & \frac{c^{RRR}_{\psi^c\psi Zh}}{\bar{\Lambda}^2}[\mathbf{13}][\mathbf{23}] + \frac{[\mathbf{12}]}{\bar{\Lambda}^3}\langle\mathbf{3}\ \{c^{RR0_A}_{\psi^c\psi Zh}(\mathbf{1}+\mathbf{2}) + c^{RR0_S}_{\psi^c\psi Zh}(\mathbf{1}-\mathbf{2})\}\ \mathbf{3}] \\
& + \frac{c^{RL0}_{\psi^c\psi Zh}}{\bar{\Lambda}^2}[\mathbf{13}]\langle\mathbf{23}\rangle + \frac{c^{RLR}_{\psi^c\psi Zh}}{\bar{\Lambda}^3}[\mathbf{312}\rangle[\mathbf{13}] + \frac{c^{RLL}_{\psi^c\psi Zh}}{\bar{\Lambda}^3}\langle\mathbf{321}]\langle\mathbf{23}\rangle \\
& + \frac{c^{LR0}_{\psi^c\psi Zh}}{\bar{\Lambda}^2}\langle\mathbf{13}\rangle[\mathbf{23}] + \frac{c^{LRR}_{\psi^c\psi Zh}}{\bar{\Lambda}^3}[\mathbf{321}\rangle[\mathbf{23}] + \frac{c^{LRL}_{\psi^c\psi Zh}}{\bar{\Lambda}^3}\langle\mathbf{312}]\langle\mathbf{13}\rangle \\
& + \frac{c^{LLL}_{\psi^c\psi Zh}}{\bar{\Lambda}^2}\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle + \frac{\langle\mathbf{12}\rangle}{\bar{\Lambda}^3}\langle\mathbf{3}\ \{c^{LL0_A}_{\psi^c\psi Zh}(\mathbf{1}+\mathbf{2}) + c^{LL0_S}_{\psi^c\psi Zh}(\mathbf{1}-\mathbf{2})\}\ \mathbf{3}]
\end{aligned}
$$

where $c_{\psi^c\psi Zh}$'s are expansions in $\tilde{s}_{ij} \equiv 2p_i\cdot p_j$

(negative power of $p_i\cdot p_j$ would cover factorizable contributions,
non-rational functions would cover loop contributions)

- Perturbative unitarity up to $\bar{\Lambda} \gg m$ forbids e.g. $[\mathbf{13}]\langle\mathbf{23}\rangle/m_3\bar{\Lambda}$.
- For $p_h \to 0$ (aka *soft Higgs limit*), one recovers $\psi^c\psi Z$ amplitudes.

Factorizable terms



Leading high-energy amplitudes $\sim E/m$:

$$(--0):\quad -\frac{\langle 12\rangle}{\sqrt{2}m_Z}\left(c^R_{\psi^c\psi Z}-c^L_{\psi^c\psi Z}\right)\left(c_{ZZh}\,\frac{m_\psi}{2m_Z}-c^L_{\psi^c\psi h}\right)$$

$$(++0):\quad +\frac{\langle 12\rangle}{\sqrt{2}m_Z}\left(c^R_{\psi^c\psi Z}-c^L_{\psi^c\psi Z}\right)\left(c_{ZZh}\,\frac{m_\psi}{2m_Z}-c^R_{\psi^c\psi h}\right)$$

either vector-like fermion: $c^R_{\psi^c\psi Z}=c^L_{\psi^c\psi Z}$

or Higgs mechanism: $c^L_{\psi^c\psi h}=c_{ZZh}\frac{m_\psi}{2m_Z}=c^R_{\psi^c\psi h}$

up to $\mathcal{O}(m/\bar{\Lambda})$

The on-shell SM EFTs

Going on-shell avoids gauge and field-redefinition redundancies.

Massless contact terms can be bootstrapped
and replace massless operator enumerations.

Massive contact terms can be constructed systematically,
although $n>4$ and spins>1 are presently cumbersome.

Gauge symmetries emerge from perturbative unitarity,
for a given particle spectrum.

The machinery is in place for massive SM EFTs applications!