

# Gravitational wave signature from a second-order Peccei-Quinn phase transition

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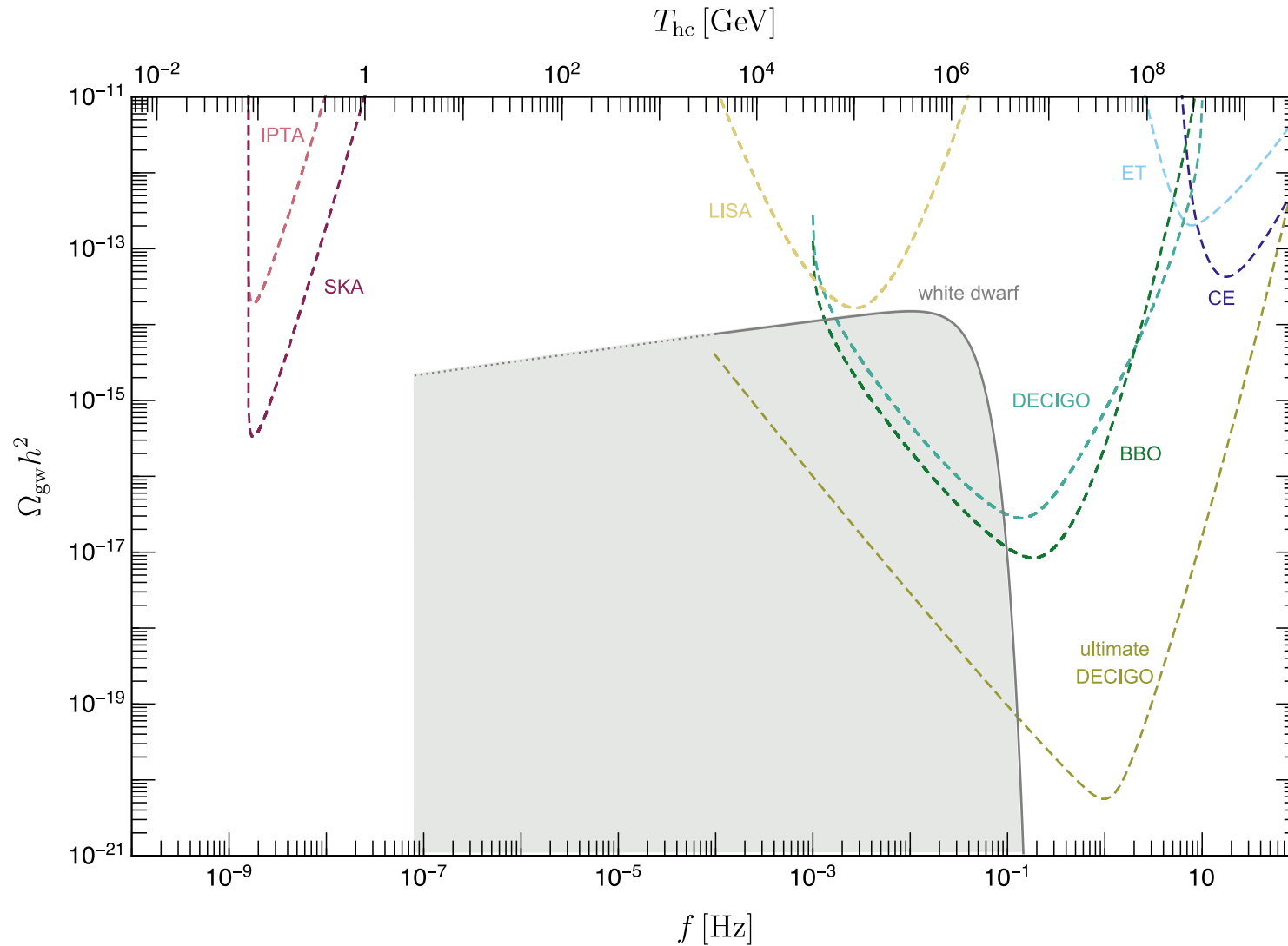
2009.02050 [hep-ph]

in collaboration with...

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DESY

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Kanazawa

# The experimental context



## The aim:

Obtain experimental predictions for features in the spectrum of **primordial gravitational** waves in the **SMASH** model associated with the **2nd-order PQ transition**

## The novelty:

Focus on 2<sup>nd</sup>-order BSM transition, rather than 1<sup>st</sup> order.

**Improved formalism** for following  $g_*$  during the phase transition

Our predictions set a **target** for the **DECIGO** experiment

## The plan:

SMASH theory and its motivation

Primordial gravitational waves: from inflation until today

Calculation of  $g_*$

Current spectrum of gravitational waves

# SMASH model and its motivation

# Current paradigm and open questions

Paradigm from cosmological data:  $\Lambda$ CDM model with an early period of **inflation**:

SM + dark matter + cosmological constant + inflationary sector.

Open questions addressed in SMASH [Ballesteros, Redondo, Ringwald, CT]

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mechanism of inflation

dark matter

baryogenesis

Higgs stability

Smallness of  $\nu$  masses

Strong CP problem

# All those problems... all those solutions

<b>Inflation</b>	Scalar inflaton
<b>Higgs stability</b>	Scalar interactions
<b>Small neutrino masses</b>	Seesaw models, radiative mass generation
<b>CP problem*</b>	Axion, Nelson-Barr
<b>Dark matter</b>	WIMP, sterile neutrinos, axion
<b>Baryogenesis</b>	Electroweak baryogenesis, leptogenesis, Affleck-Dine...

\*See however [arXiv:2001.07152](https://arxiv.org/abs/2001.07152) [hep-th]

# S.M.A.S.H

**Minimal SM extension** providing a **consistent, predictive picture** of:

Particle physics from the electroweak to the Planck scale

Cosmology from inflation to today

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## Highlights:

**Single new scale**, playing a role in stability, the CP problem, neutrino masses, dark matter, and baryogenesis

**Predictive inflation** free from unitarity concerns

Detailed understanding of **parameter space** yielding **stability**

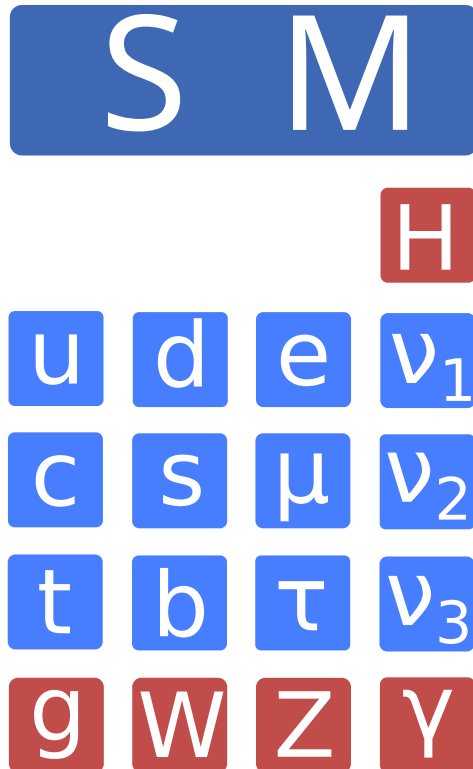
Detailed understanding of **reheating** and **post-inflationary history**

Accurate **predictions** for **cosmological parameters** and the **axion mass** in reach of future experiments



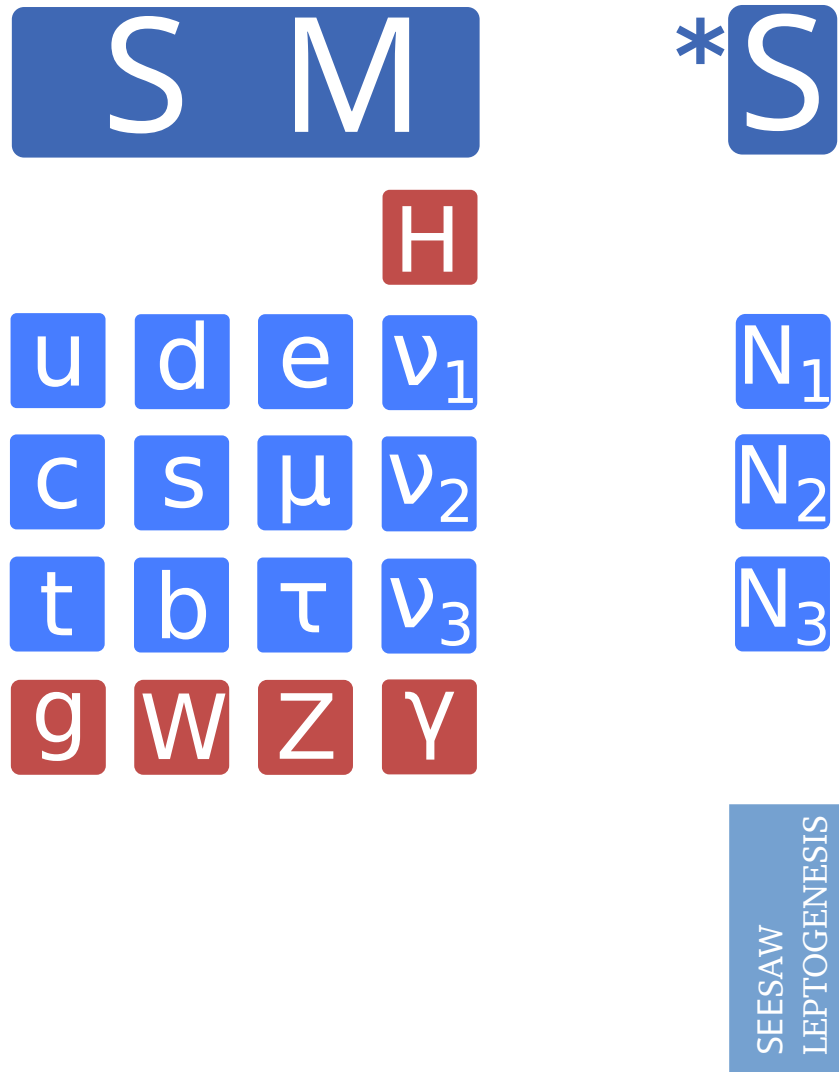
# Building up SMASH

6 problems addressed with 3 new types of particles.



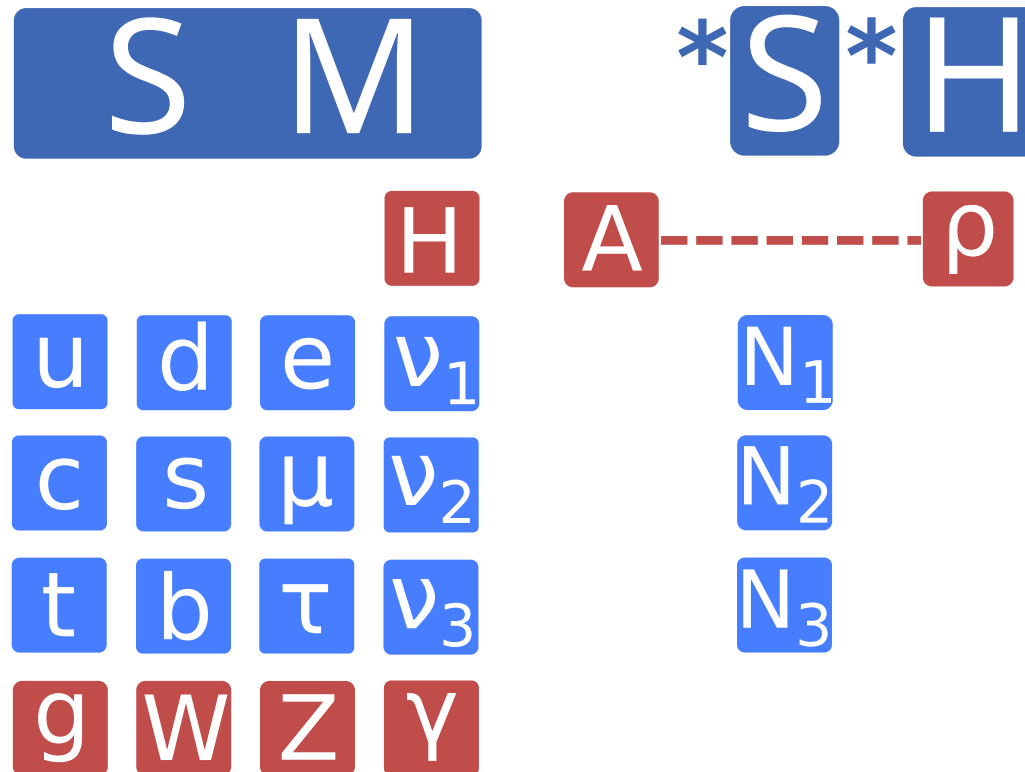
# Building up SMASH

Start with right handed neutrinos, addressing  $\nu$  masses and baryogenesis.



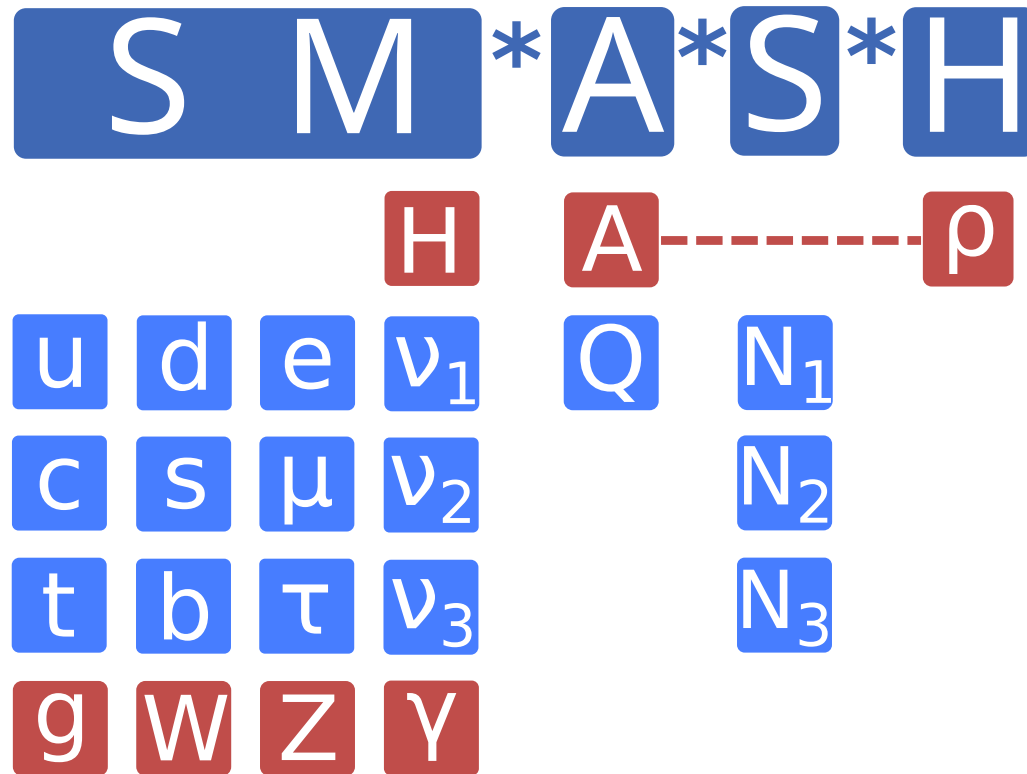
# Building up SMASH

Add a new scalar  $\sigma$  to provide inflation. As a bonus, it can stabilize the Higgs, more so if it gets a VEV! (threshold mechanism [Lebedev, Elias-Miró et al])



# Building up SMASH

Singlet scalar with VEV can implement the KSVZ axion solution to the CP problem.  
 Need a Dirac fermion in the fundamental of SU(3). Bonus: axion can be dark matter!



CP PROBLEM  
 DARK MATTER

SEESAW  
 LEPTOGENESIS

INFLATION  
 STABILITY

# SMASH recap

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{yuk}}^{SM}$$

$$- \left[ \frac{M^2}{2} + \xi_H H^\dagger H + \xi_\sigma |\sigma|^2 \right] R$$

INFLATION

$$- \lambda_H \left( H^\dagger H - \frac{v^2}{2} \right)^2 - 2\lambda_{H\sigma} \left( H^\dagger H - \frac{v^2}{2} \right) \left( |\sigma|^2 - \frac{v_\sigma^2}{2} \right)$$

STABILITY

$$- \lambda_\sigma \left( |\sigma|^2 - \frac{v_\sigma^2}{2} \right)^2 - [y\sigma\tilde{Q}Q + y_{Q_{d_i}}\sigma Qd_i + c.c.]$$

CP, DARK MATTER

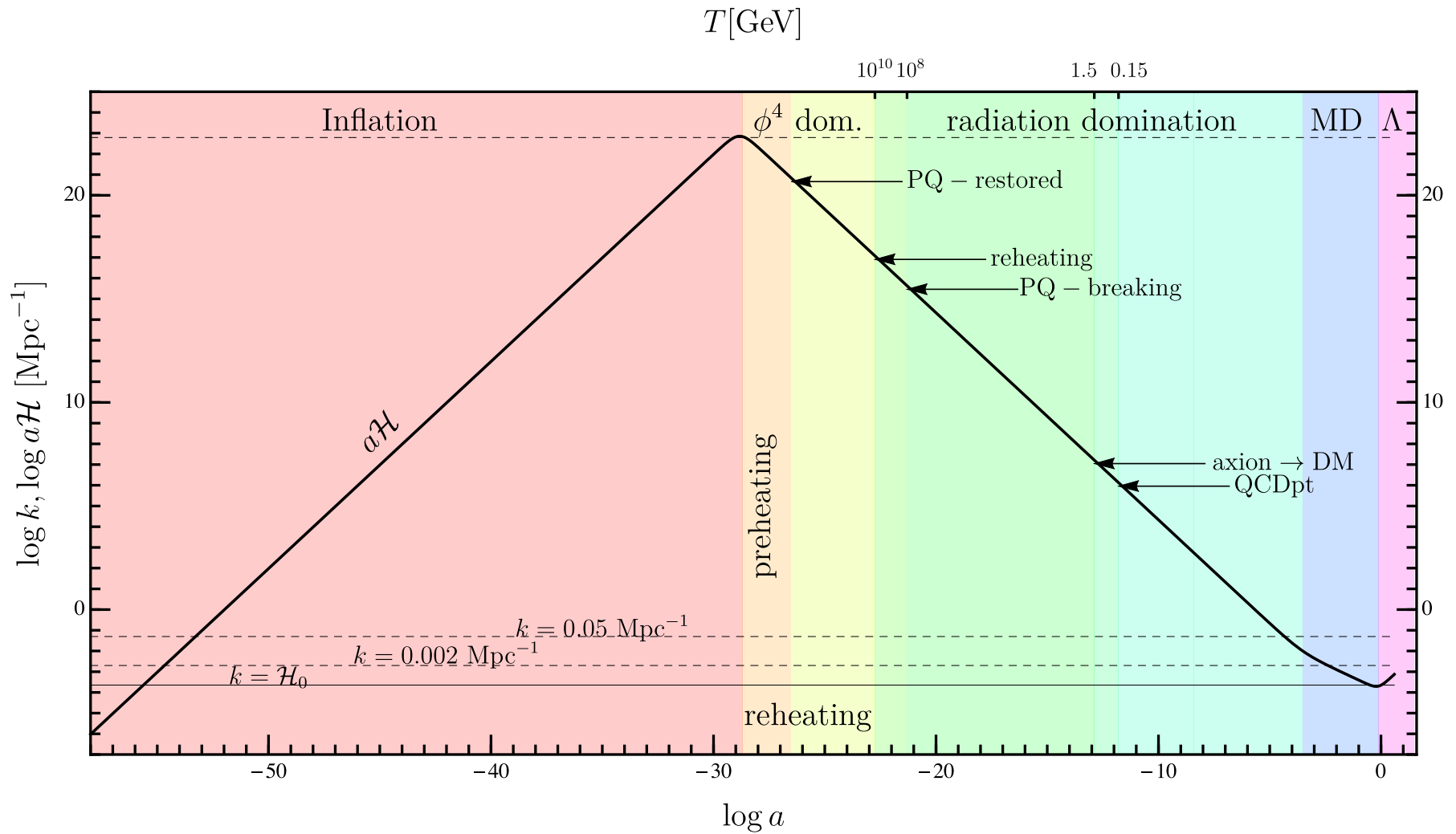
$$- [F_{ij}L_i\epsilon HN_j + \frac{1}{2}Y_{ij}\sigma N_i N_j + c.c.]$$

SEESAW AND LEPTOGENESIS

Most general, renormalizable Lagrangian compatible with the following global PQ symmetry:

$q$	$u$	$d$	$L$	$N$	$E$	$Q$	$\tilde{Q}$	$\sigma$
1/2	-1/2	-1/2	1/2	-1/2	-1/2	-1/2	-1/2	1

# SMASHy history of the Universe



# Preferred parameter choices

From **inflation** and **unitarity**:

$$5 \times 10^{-13} \lesssim \lambda_\sigma \lesssim 5 \times 10^{-10}$$

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From **Higgs stability**:

$$\lambda_{H\sigma} \sim 0.3\sqrt{\lambda_\sigma}$$

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From **stability** of  $\sigma$ :

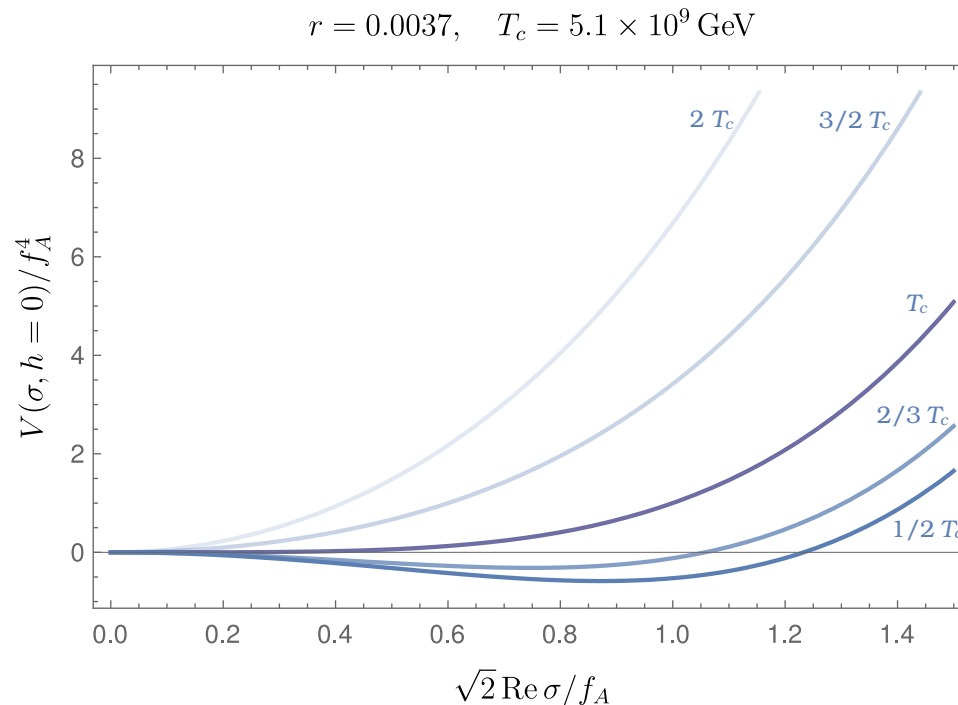
$$Y_{ij}, y \lesssim \sqrt{\lambda_\sigma}$$

# PQ phase transition in SMASH

PQ phase transition predicted around a **particular window of temperatures**

$$T_c \simeq \frac{2\sqrt{6\lambda_\sigma}v_\sigma}{\sqrt{8(\lambda_\sigma + \lambda_{H\sigma}) + \sum_i Y_{ii}^2 + 6y^2}} \sim \lambda_\sigma^{1/4}v_\sigma \sim \mathcal{O}(10^6 - 10^9) \text{ GeV}$$

Phase transition is **second order**





# Gravitational waves from inflation

$$ds^2 \supset -dt^2 + a^2(t)(\delta_{ij} - 2\mathcal{R} + h_{ij})dx^i dx^j,$$

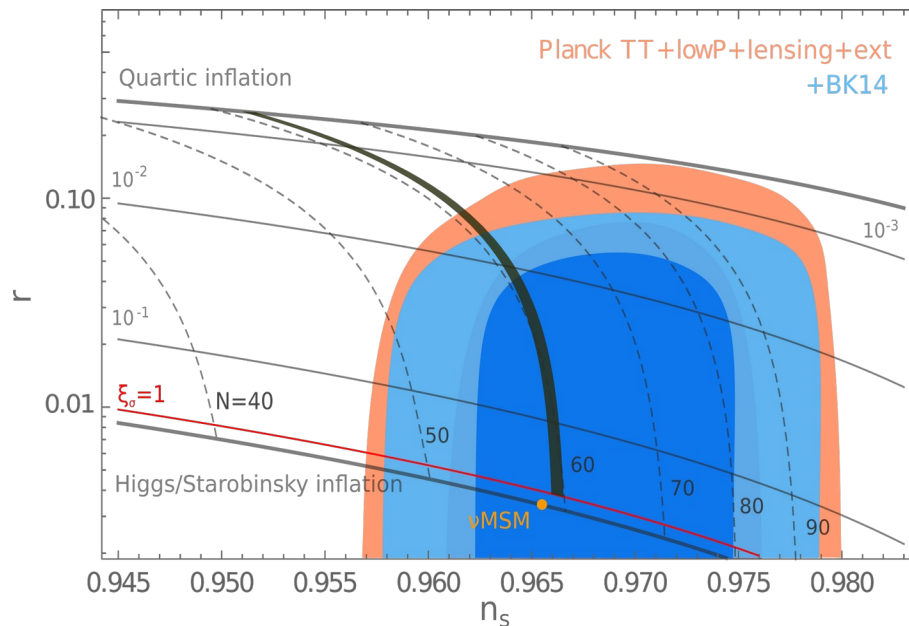
$$\langle \mathcal{R}(t, \vec{x}) \mathcal{R}(t, \vec{x}') \rangle = \int \frac{dk}{k} \Delta_{\mathcal{R},k}^2 \quad \Delta_{\mathcal{R},k,\text{prim}}^2 \approx A_s(k_*) \left( \frac{k}{k_*} \right)^{n_s(k_*)-1}$$

spectral index

$$\sum_{ij} \langle h_{ij}(t, \vec{x}) h_{ij}(t, \vec{x}') \rangle = \int \frac{dk}{k} \Delta_{T,k}^2(t) \quad \Delta_{T,k,\text{prim}}^2 \approx r A_s(k_*) \left( \frac{k}{k_*} \right)^{-r/8}$$

tensor-to-scalar ratio

current bound  $r < 0.058$



Possible CORE  $n_s$  resolution

# Gravitational waves from the PQ transition?

Second-order phase **transition** proceeds adiabatically, **without** further **breaking** of spatial **homogeneity**

Sourcing gravitational waves requires spatial anisotropies (quadrupole contributions to energy-momentum tensor)

Thus the **PQ phase transition** does not source new gravitational waves, but it **affects the propagation of primordial waves** generated during inflation

This is in contrast to first-order phase transitions proceeding through bubble nucleation and sourcing gravitational waves

Primordial gravitational waves:  
from inflation until today

# Thermodynamics during radiation domination

Energy density and entropy during radiation domination:

$$\rho = \left. \frac{\partial U}{\partial V} \right|_T \equiv \frac{\pi^2}{30} g_{*\rho} T^4 \qquad s = \left. \frac{\partial S}{\partial V} \right|_T \equiv \frac{2\pi^2}{45} g_{*s} T^3$$

Both related to pressure from thermodynamical identities

$$s = \frac{\partial p}{\partial T} \qquad dU = TdS - pdV \Rightarrow \rho = Ts - p = T \frac{\partial p}{\partial T} - p$$

Free-energy density  $\longleftrightarrow$  finite  $T$  effective potential  $\longleftrightarrow$  pressure

$$A = U - TS \Rightarrow \left. \frac{\partial A}{\partial V} \right|_T = V_{\text{eff},\min}(T, \bar{\phi}_i) - V_{\text{eff}}(0, \bar{\phi}_i) \equiv \Delta V(T) = -p$$

$$g_{*\rho} = \frac{30}{\pi^2 T^4} \left( \Delta V(T) - T \frac{\partial \Delta V(T)}{\partial T} \right),$$
$$g_{*s} = - \frac{45}{2\pi^2 T^3} \frac{\partial \Delta V(T)}{\partial T}.$$

Can be computed directly from finite  $T$  effective potential!

# Metric perturbations and power spectrum

$$\begin{aligned}
 ds^2 &\supset -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j, \\
 h_i^i &= 0 \\
 \partial^i h_{ij} &= 0
 \end{aligned}$$

$$h_{ij}(t, \mathbf{x}) = \sum_{\lambda=+, \times} \int \frac{d^3k}{(2\pi)^3} (h_{\mathbf{k}}^\lambda(t) \epsilon_{ij}^\lambda(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} \mathbf{a}_{\mathbf{k}}^\lambda + h_{\mathbf{k}}^{\lambda*}(t) \epsilon_{ij}^{\lambda*}(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}} \mathbf{a}_{\mathbf{k}}^{\lambda\dagger}),$$

$$\sum_{ij} \epsilon_{ij}^\lambda \epsilon_{ij}^{\lambda*} = 2\delta^{\lambda\lambda'}$$

$$[\mathbf{a}_{\mathbf{k}}^\lambda, \mathbf{a}_{\mathbf{k}'}^{\lambda'}] = (2\pi)^3 \delta^{\lambda\lambda'} \delta^3(\mathbf{k} - \mathbf{k}')$$

$$\sum_{ij} \langle h_{ij}(t, \vec{x}) h_{ij}(t, \vec{x}) \rangle = \sum_{\lambda} \int \frac{dk}{\pi^2} k^2 |h_{\mathbf{k}}^\lambda(t)|^2 \equiv \int \frac{dk}{k} \Delta_{T,k}^2(t)$$

# Qualitative behaviour of modes

$$\ddot{h}_{\mathbf{k}}^\lambda + 3H\dot{h}_{\mathbf{k}}^\lambda + \frac{k^2}{a^2} h_{\mathbf{k}}^\lambda = 16\pi G\Pi_{\mathbf{k}}^\lambda \quad \text{Source is zero for perfect fluid}$$

## Superhorizon:

$$\ddot{h}_{\mathbf{k}}^\lambda + 3H\dot{h}_{\mathbf{k}}^\lambda = 0 \Rightarrow h_{\mathbf{k}}^\lambda = \text{constant} \equiv h_{\mathbf{k},\text{prim}}^\lambda$$

Modes frozen between horizon crossing in inflation and horizon reentry RD

## Subhorizon during radiation domination:

$$\ddot{h}_{\mathbf{k}}^\lambda + \frac{k^2}{a^2} h_{\mathbf{k}}^\lambda = 0 \Rightarrow h_{\mathbf{k}}^\lambda \propto \frac{e^{ik\sqrt{t}}}{a}$$

$$h_{\mathbf{k}}(t) \equiv h_{\mathbf{k},\text{prim}} \chi_k \approx h_{\mathbf{k},\text{prim}} e^{ik(\tau - \tau_{\text{hc}})} \frac{a(t_{\text{hc}})}{a(t)}$$

To leading order, power spectrum at late times simply obtained by redshifting inflationary power spectrum

# From power spectrum to energy density

$$\rho_{\text{gw}} = \frac{M_P^2}{4} \langle \dot{h}_{ij}(t, \mathbf{x}) \dot{h}_{ij}(t, \mathbf{x}) \rangle$$

$$\rho_{\text{crit}} = 3H^2 M_P^2$$

$$\Omega_{\text{gw}} = \frac{\rho_{\text{gw}}}{\rho_{\text{crit}}} \equiv \int \frac{dk}{k} \Omega_{\text{gw}}(k)$$

$$\Omega_{\text{gw}}(k) = \frac{1}{12H^2} \frac{k^3}{\pi^2} \sum_{\lambda} |\dot{h}_{\mathbf{k}}(t)|^2 = \frac{1}{12H^2 a^2} \Delta_{T,k,\text{prim}}^2 |\chi'_{\mathbf{k}}(t)|^2 \approx \frac{1}{24} \Delta_{T,k,\text{prim}}^2 \frac{a^4(\tau_{\text{hc}}) H^2(\tau_{\text{hc}})}{a^4(\tau) H^2(\tau)}$$

$$' \equiv \frac{d}{d\tau} = a \frac{d}{dt}$$

$$h_{\mathbf{k}} = h_{\mathbf{k},\text{prim}} \chi_{\mathbf{k}}$$

$$\chi_{\mathbf{k}} \approx e^{ik(\tau - \tau_{\text{hc}})} \frac{a(\tau_{\text{hc}})}{a(\tau)}$$

$$k \equiv a(\tau_{\text{hc}}) H(\tau_{\text{hc}}) \gg aH$$

$$\Omega_{\text{gw}}(k) \approx \frac{1}{24} \Delta_{T,k,\text{prim}}^2 \Omega_{\gamma} \left( \frac{g_{*\rho,\text{hc}}}{2} \right) \left( \frac{g_{*s,\text{hc}}}{g_{*s,0}} \right)^{-4/3}$$

$$H^2 = \frac{\rho}{3M_P^2}$$

$$S \propto sa(\tau)^3 \propto g_{*s} T^3 a^3 = \text{const}$$

$$\rho = \frac{\pi^2}{30} g_{*\rho} T^4$$

# From power spectrum to energy density

$$\Omega_{\text{gw}}(k) \approx \frac{1}{24} \Delta_{T,k,\text{prim}}^2 \Omega_\gamma \left( \frac{g_{*\rho,\text{hc}}}{2} \right) \left( \frac{g_{*s}(T_{\text{hc}}(k))}{g_{*s,0}(k)} \right)^{-4/3}$$

$$\Delta_{T,k,\text{prim}}^2 = \frac{2H_{\text{inf}}^2}{\pi^2 M_{\text{P}}^2} \Big|_{k=a_{\text{inf}} H_{\text{inf}}} \approx r A_s(k_*) \left( \frac{k}{k_*} \right)^{-r/8} \quad r < 0.058$$

Almost scale-invariant spectrum

Sudden changes in  $g_{*\rho}, g_{*s}$  can lead to **steps in power spectrum**

This happens in **phase transitions!**

[Schwarz, Watanabe & Komatsu, Boyle & Steinhard, Saikawa & Shirai]



# Remarks on second-order versus first-order

## First order phase transition

Sources new gravitational waves  
from expanding and colliding bubbles

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## Second order phase transition

Leads to steps in power spectrum of primordial  
gravitational waves

Does the PQ transition in SMASH lead to observable signatures?

# The spectrum time machine

$$f = \frac{k}{2\pi a_0} = \frac{g_{*\rho}^{1/2}(T_{\text{hc}})}{2\sqrt{90}} \left( \frac{g_{*s}(T_0)}{g_{*s}(T)} \right)^{1/3} \frac{T_0 T_{\text{hc}}}{M_P}$$

Higher frequencies crossed the horizon earlier

By looking at higher  $f$  we probe how the universe was at earlier and earlier times!

Spectrum with  $f > 0.1$  Hz above white-dwarf noise actually probes  $T > 10^6$  GeV

# Going beyond simplest picture

Previous calculations ignored source effects

**Free-streaming particles** source anisotropies in the stress-energy tensor, which contribute to **source in wave equation** [Weinberg]

$$h_{\mathbf{k}}(t) \equiv h_{\mathbf{k},\text{prim}} \chi_{\mathbf{k}} \quad u = k\tau$$

$$\frac{d^2 \chi(u)}{du^2} + \frac{2}{a(u)} \frac{da(u)}{du} \frac{d\chi(u)}{du} + \chi(u) = -24 \sum_{i=\gamma,\nu,a} F_i(u) \left[ \frac{1}{a(u)} \frac{da(u)}{du} \right]^2 \int_{u_i}^u dU \left[ \frac{j_2(u-U)}{(u-U)^2} \right] \frac{d\chi(U)}{dU},$$

$$F_i(u) \equiv \frac{\rho_i(u)}{\rho_{\text{crit}}(u)}$$

Time at which species  $i$  starts to free-stream

Still need  $g_{*p}, g_{*s}$  to relate  $u$  with temperature and compute  $F_i(u)$

# The plan

Precise calculation of  $g_{*\rho}, g_{*s}$  throughout the PQ phase transition in SMASH

Solve the differential equations for  $\chi$  including the effect of **free-streaming** photons, neutrinos and relativistic axions

Calculation of  $g^*$  and  $g^*_s$

# Main features of our calculation

**Full one-loop** finite T potential with **improved Daisy resummation** of thermal self-energies of  $H, \sigma, B_\mu$

**3 loop QCD corrections** to pressure

Corrections from **axion decoupling**

# Finite temperature effective potential

$$g_{*\rho} = \frac{30}{\pi^2 T^4} \left( \Delta V(T) - T \frac{\partial \Delta V(T)}{\partial T} \right),$$
$$g_{*s} = - \frac{45}{2\pi^2 T^3} \frac{\partial \Delta V(T)}{\partial T}.$$

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Finite  $T$  captured at **leading order** by  $\Delta m_\sigma^2(T) \sim \text{coupling}^\kappa T^2$

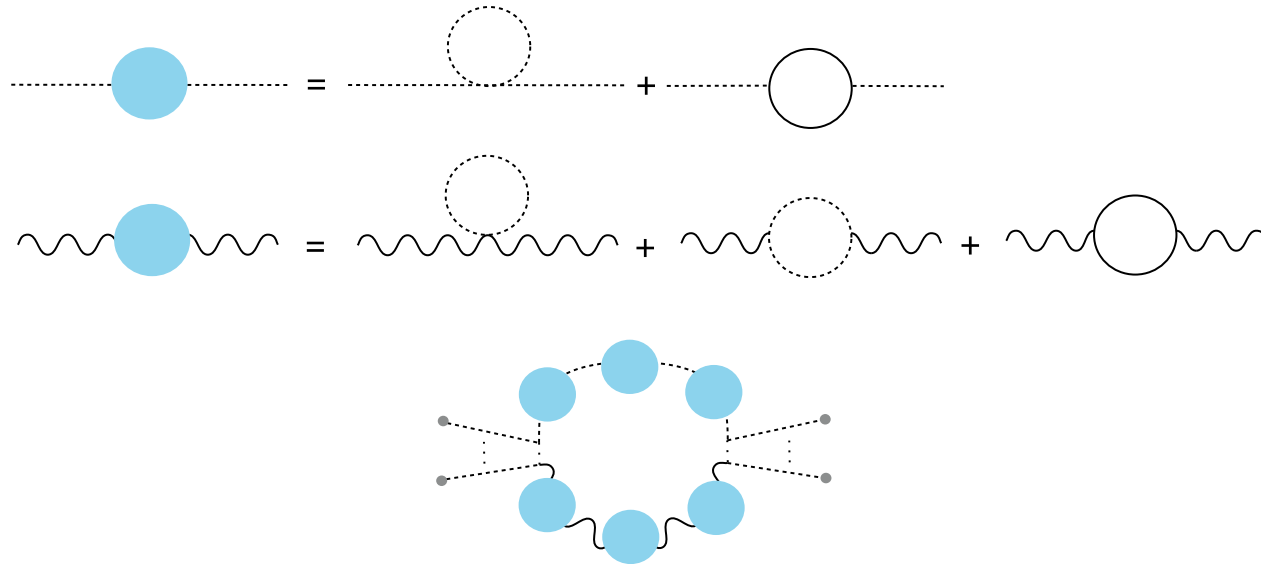
Thermal  **$N$ -loop corrections** from **bosonic self-energies** go as

$$(\text{0 T result}) \times \left( \frac{\text{coupling}^\kappa T^2}{m_\sigma^2} \right)^N$$

**Phase transition** happens when  $\Delta m_\sigma^2(T) \sim m_\sigma^2$  : **all loop corrections similar!**

**Need to resum**  $N$ -loop effects: **Daisy resummation** of bosonic self-energies

# The trouble with the usual Daisys



Usually, the **leading order** contribution a **high- $T$  expansion** is taken:

$$\text{---} \bullet \text{---} , \text{~} \text{---} \bullet \text{~} \sim \text{coupling}^\kappa T^2$$

Phase transition makes some particles **massive**

**Massive particles decouple** from thermal plasma

Usual **Daisy resummation incompatible with decoupling**: overestimates  $g_{*\rho}, g_{*s}$



# Improved resummation

Instead of a high- $T$  expansion, we capture the **full  $T$  dependence at zero momentum**

We apply improved resummation to contributions to bosonic self-energies from **particles that get heavy during PQ phase transition**

**Particles getting heavy:**

$$\sigma, Q, \tilde{Q}, N_i$$

**Contribute to self-energies of**

$$H, \sigma, B_\mu$$

Improved resummation

$$G_\mu^a, \quad a = 1, \dots, 8$$

Alternate treatment

# Improved resummation

Improved resummation can be captured by mass corrections that do not go as  $T^2$

**Corrected mass of a scalar** coupling to heavy scalars and fermions:

$$\Delta m_{\phi_j}^2(\bar{\phi}_i, T) \supset \sum_B \frac{T^2}{\pi^2} J'_B \left( \frac{m_B^2(\bar{\phi}_i)}{T^2} \right) \frac{\partial m_B^2(\phi_i)}{\partial \phi_j^2} \Big|_{\phi_i \rightarrow \bar{\phi}_i} - 2 \sum_F \frac{T^2}{\pi^2} J'_F \left( \frac{m_F^2(\bar{\phi}_i)}{T^2} \right) \frac{\partial m_F^2(\phi_i)}{\partial \phi_j^2} \Big|_{\phi_i \rightarrow \bar{\phi}_i}$$

**Corrected mass of gauge field** coupling to heavy fermions;

$$\Delta m_G^2(\bar{\phi}_i, T) \supset \sum_F \frac{\tilde{g}^2 q_F^2 T^2}{\pi^2} K_F \left( \frac{m_F^2(\bar{\phi}_i)}{T^2} \right)$$

$$J_B(x) = \int_0^\infty dy y^2 \log \left[ 1 - \exp(-\sqrt{x+y^2}) \right], \quad J_F(x) = \int_0^\infty dy y^2 \log \left[ 1 + \exp(-\sqrt{x+y^2}) \right].$$

$$K_F(x) = \int_0^\infty dy \frac{y^2 e^{\sqrt{x+y^2}}}{(e^{\sqrt{x+y^2}} + 1)^2}.$$

# 3 loop QCD corrections to $\Delta V$

QCD corrections to  $\Delta V$  are known to **3 loop** order in a theory with **arbitrary massless fermionic flavours** [Kajantie et al]

Using the former with the improved Daisy resummation would incur into **double-counting**

We implement the **decoupling** of  $Q, \tilde{Q}$  by **interpolating** in temperature between SM 6 flavour result and SMASH 7 flavour result, weighing with thermal loop functions

$$\Delta V^{QCD}(T) = \left(1 - \frac{J_F(m_Q^2/T)}{J_F(0)}\right) \Delta V_{6 \text{ flavours}}^{QCD}(T) + \frac{J_F(m_Q^2/T)}{J_F(0)} \Delta V_{7 \text{ flavours}}^{QCD}(T)$$

# Assembling pieces

$$V_{\text{eff}}(\sigma, T) = V(H = 0, \sigma) + V^{\text{CW}}(\sigma, T) + \Delta V^T(\sigma, T) + V^{\text{QCD}}(T).$$

$$V^{\text{CW}}(\sigma, T) = \frac{1}{64\pi^2} \left[ \sum_V m_V^4(\sigma, T) \left( \log \frac{m_V^2(\sigma, T)}{\mu^2} - \frac{5}{6} \right) + \sum_S m_S^4(\sigma, T) \left( \log \frac{m_S^2(\sigma, T)}{\mu^2} - \frac{3}{2} \right) - \sum_F m_F^4(\sigma, T) \left( \log \frac{m_F^2(\sigma, T)}{\mu^2} - \frac{3}{2} \right) \right],$$

$$V^T = \frac{T^4}{2\pi^2} \left[ \sum_B J_B \left( \frac{m_B^2(\sigma, T)}{T^2} \right) - \sum_F J_F \left( \frac{m_F^2(\sigma, T)}{T^2} \right) - \sum_G J_B(0) \right],$$

**B**: Vectors (V, 3 pol. in Landau gauge) + real scalars (S)

**F**: Weyl fermions

**G**: Ghosts

2 and 3 loop QCD corrections

# Axion decoupling effects

**Axion** remains approximately massless but **loses kinetic equilibrium** with the rest of the plasma as interaction rates go as  $T^3/f_A^2$

We **approximate decoupling temperature** by  $T$  for which **trace of energy momentum tensor is maximal** (signalling completion of phase transition and emergence of PQ scale).

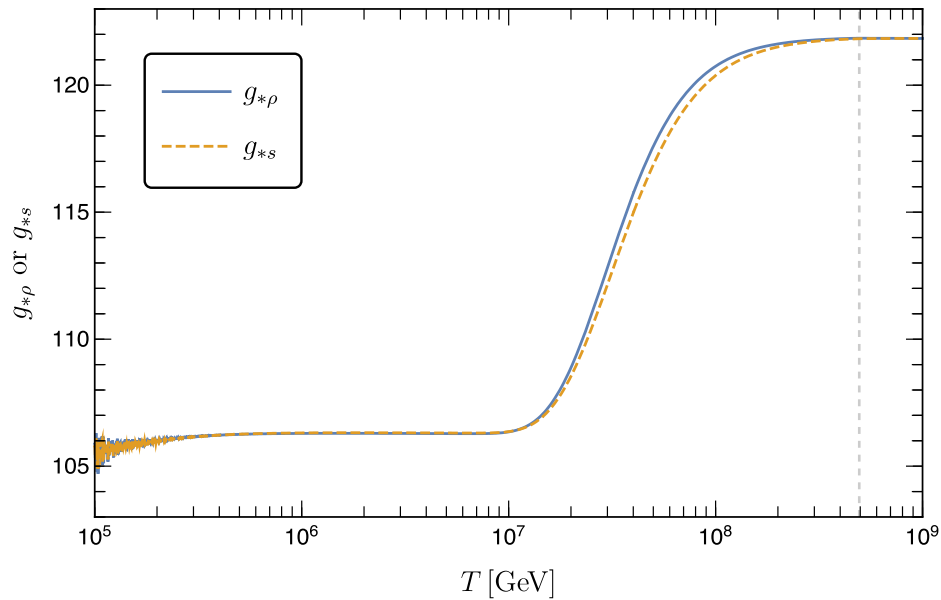
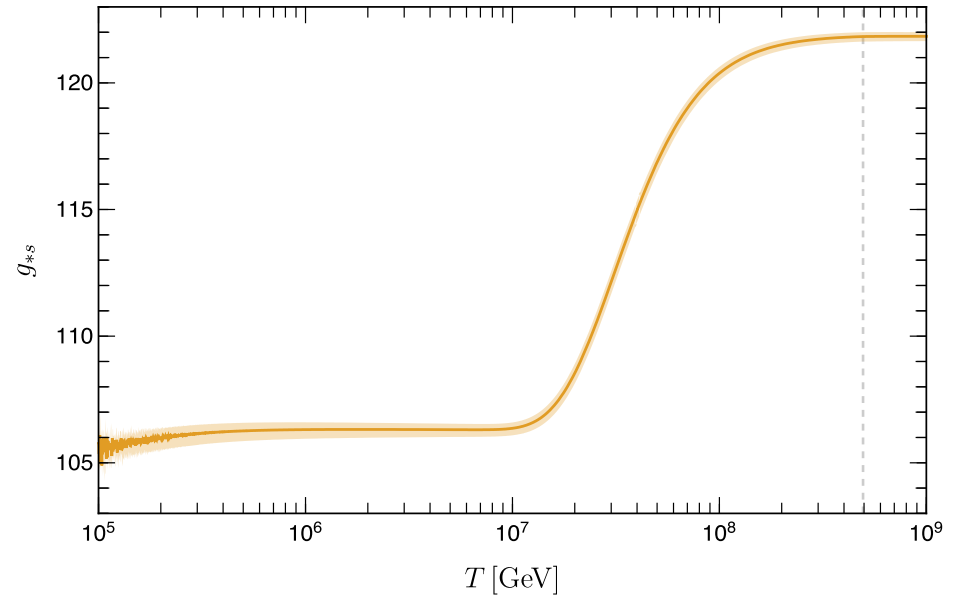
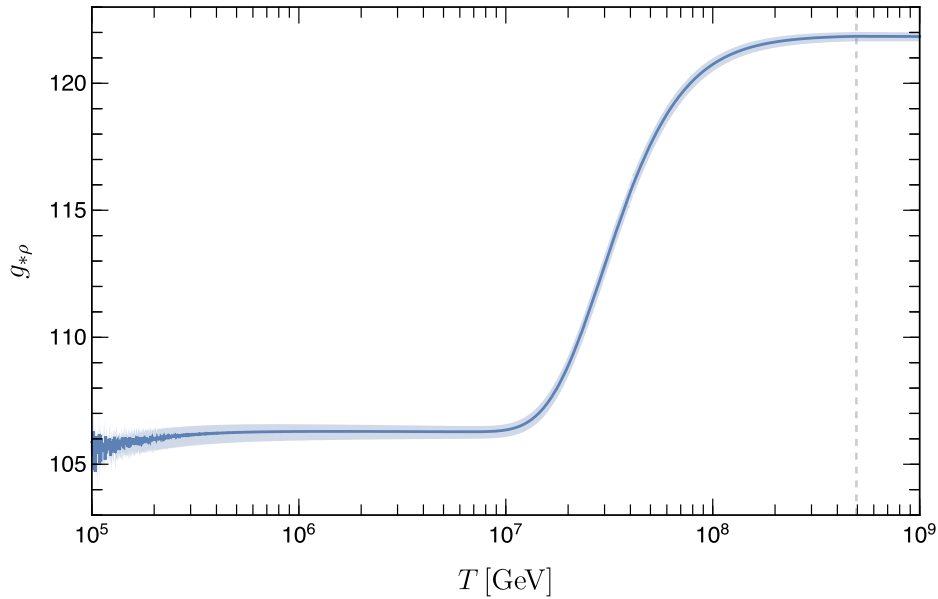
$$\Delta(T) = \frac{T^\mu_\mu}{T^4} = \frac{\rho - 3p}{T^4} = \frac{1}{T^4} \left( 4V_{\text{eff},\text{min}}(T) - T \frac{\partial V_{\text{eff},\text{min}}(T)}{\partial T} \right).$$

After decoupling, **entropies** of axion and plasma **separately conserved: axion bath has its own temperature**.

$$T_{\text{axion}} = \begin{cases} T, & T \geq T_{\text{dec}}, \\ \left( \frac{g_{*s}^{\text{bath}}(T)}{g_{*s}^{\text{bath}}(T_{\text{dec}})} \right)^{\frac{1}{3}} T, & T < T_{\text{dec}}. \end{cases}$$

$$g_{*\rho} = g_{*\rho}^{\text{eq}} - 1 + \left( \frac{T_{\text{axion}}}{T} \right)^4, \quad g_{*s} = g_{*s}^{\text{eq}} - 1 + \left( \frac{T_{\text{axion}}}{T} \right)^3$$

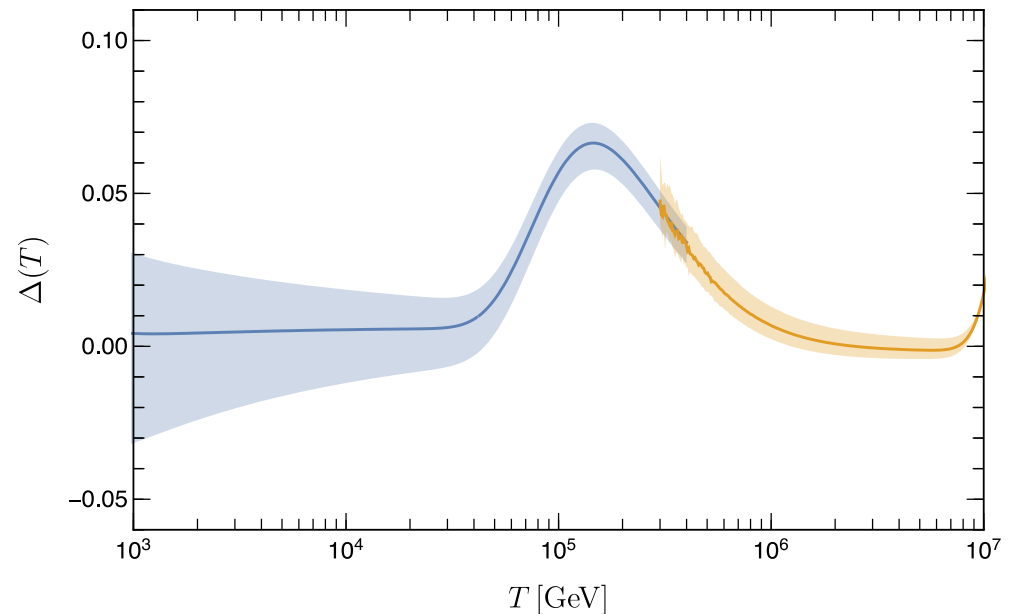
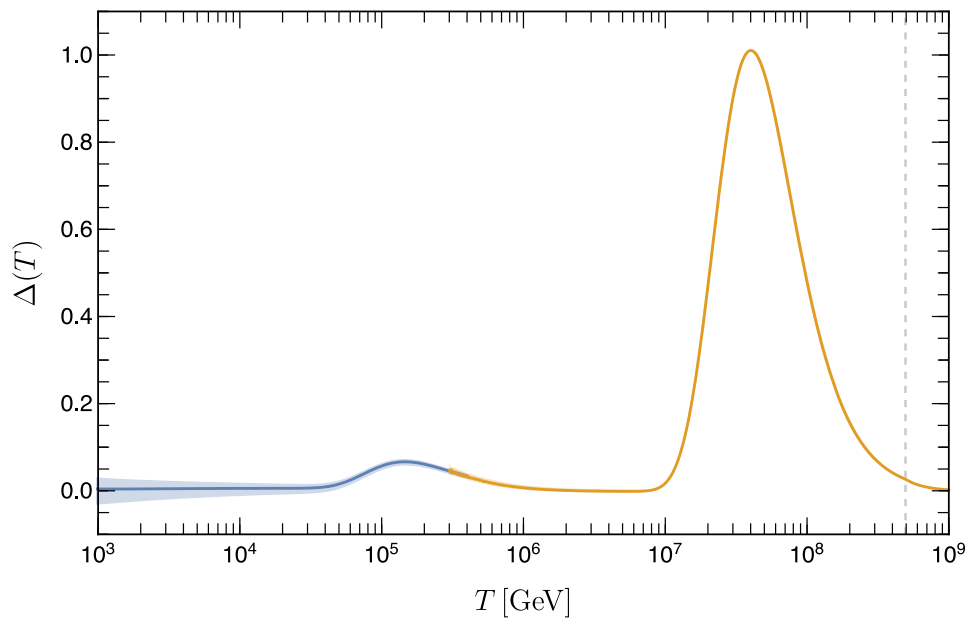
# Results during PQ phase transition



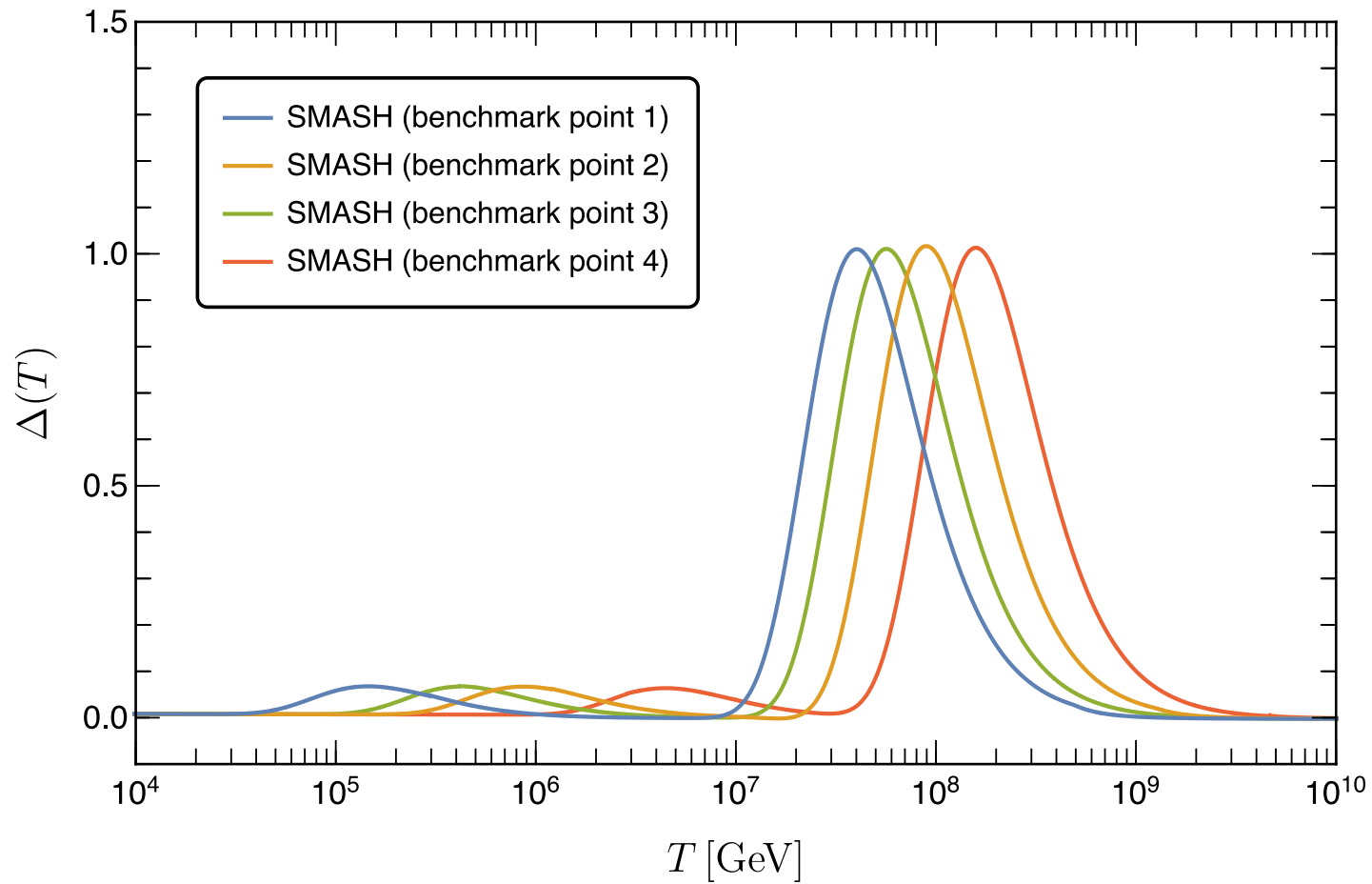
# Beyond the PQ phase transition

At lower scales we **match** our results to the **SM plus decoupled axion plus massive excitation of real part of  $\sigma$** .

For the **SM** we use results of [Shaikawa Shirai 18] including **nonperturbative lattice estimates** across the electroweak and QCD crossovers

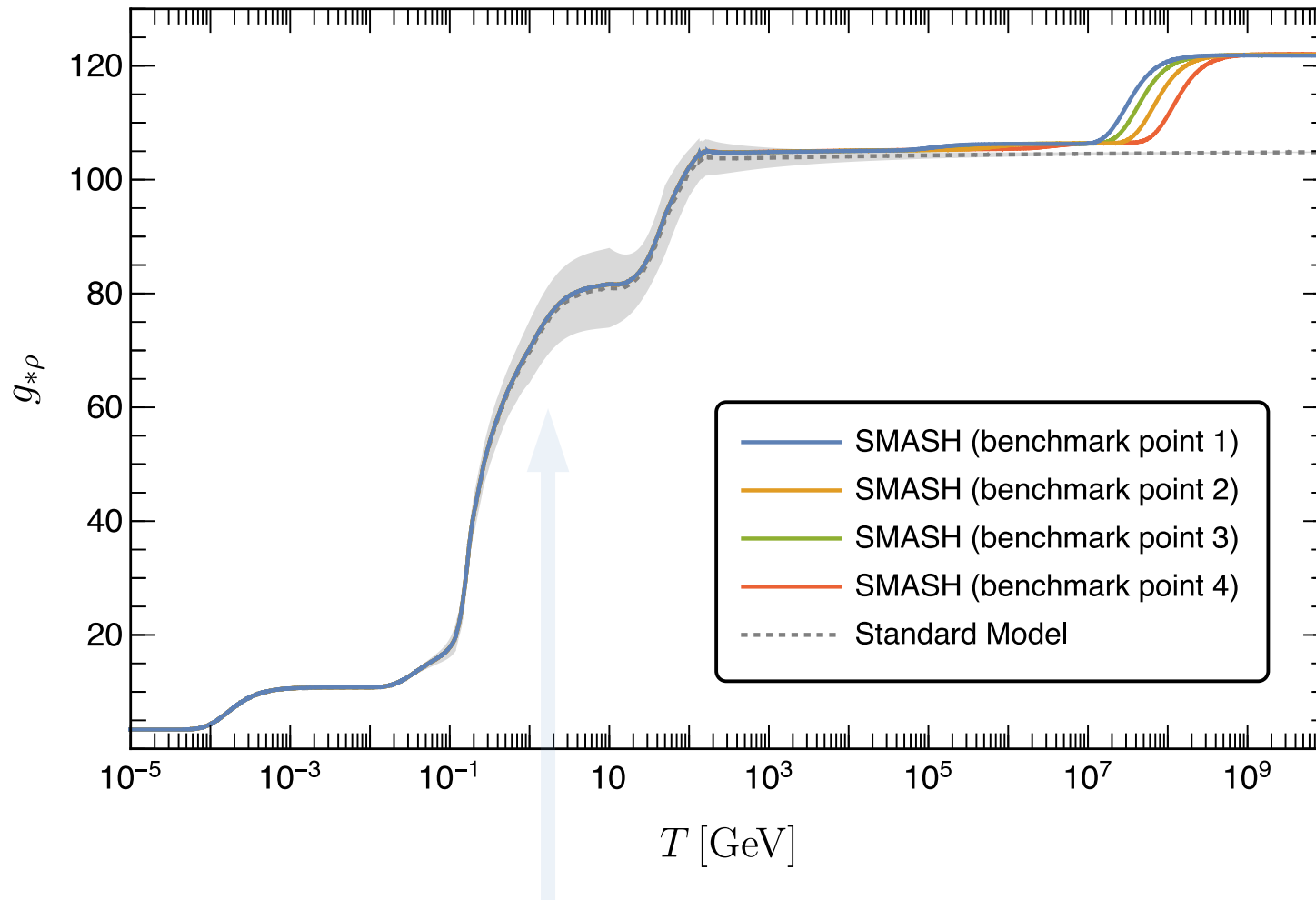


# Beyond the PQ phase transition



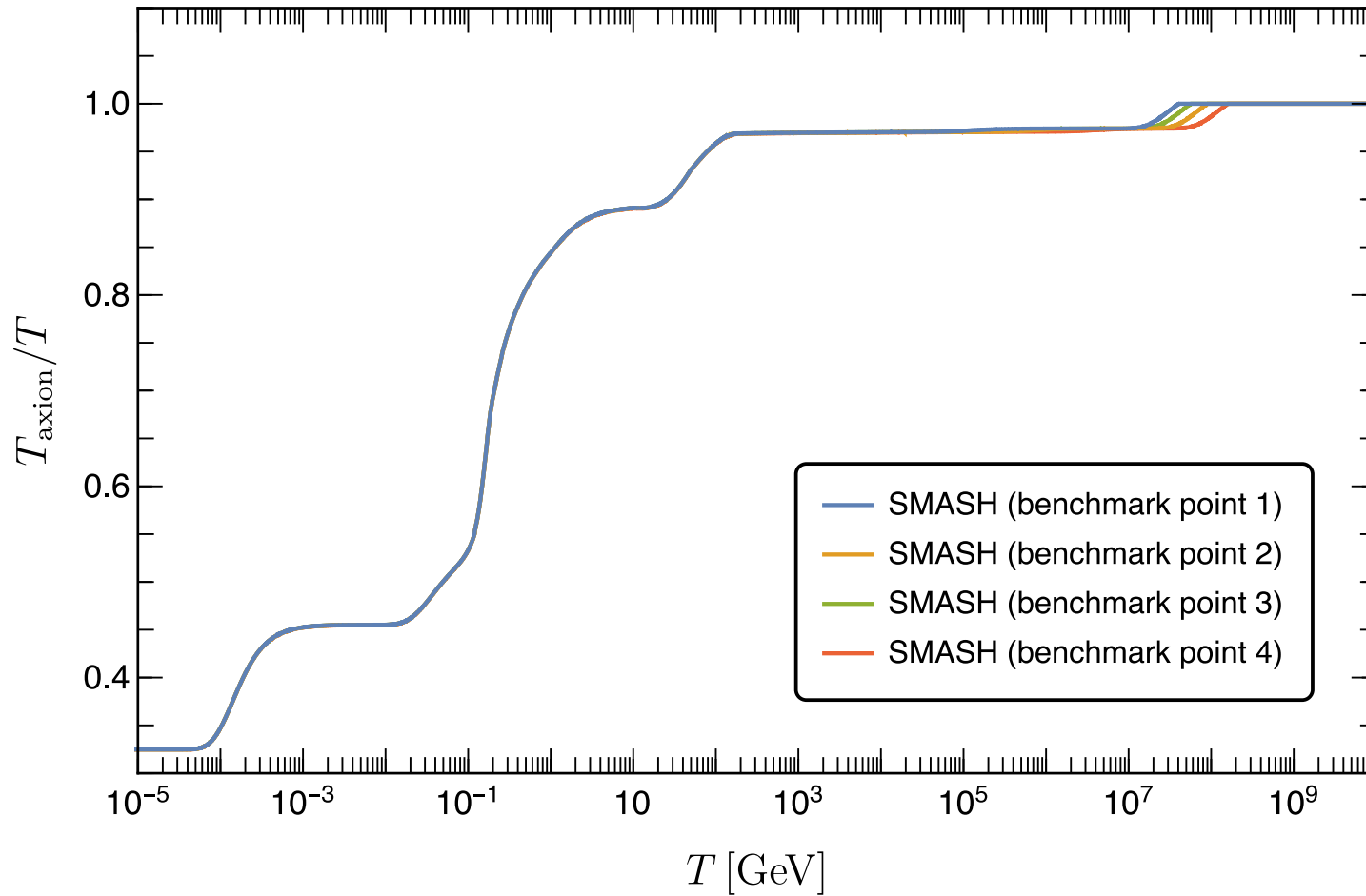


# Beyond the PQ phase transition



Bigger SM steps won't be observable because of white-dwarf noise!

# Beyond the PQ phase transition



Current spectrum of gravitational waves

# Piecing things together

$$\Omega_{\text{gw}}(k) = \frac{1}{12H^2 a^2} \Delta_{T,k,\text{prim}}^2 |\chi'_{\mathbf{k}}(t)|^2 \equiv \mathcal{T}(f) \Delta_{T,k,\text{prim}}^2$$

$$\Delta_{T,k,\text{prim}}^2 = \frac{2H_{\text{inf}}^2}{\pi^2 M_P^2} \Big|_{k=a_{\text{inf}} H_{\text{inf}}}$$

computed beyond slow roll

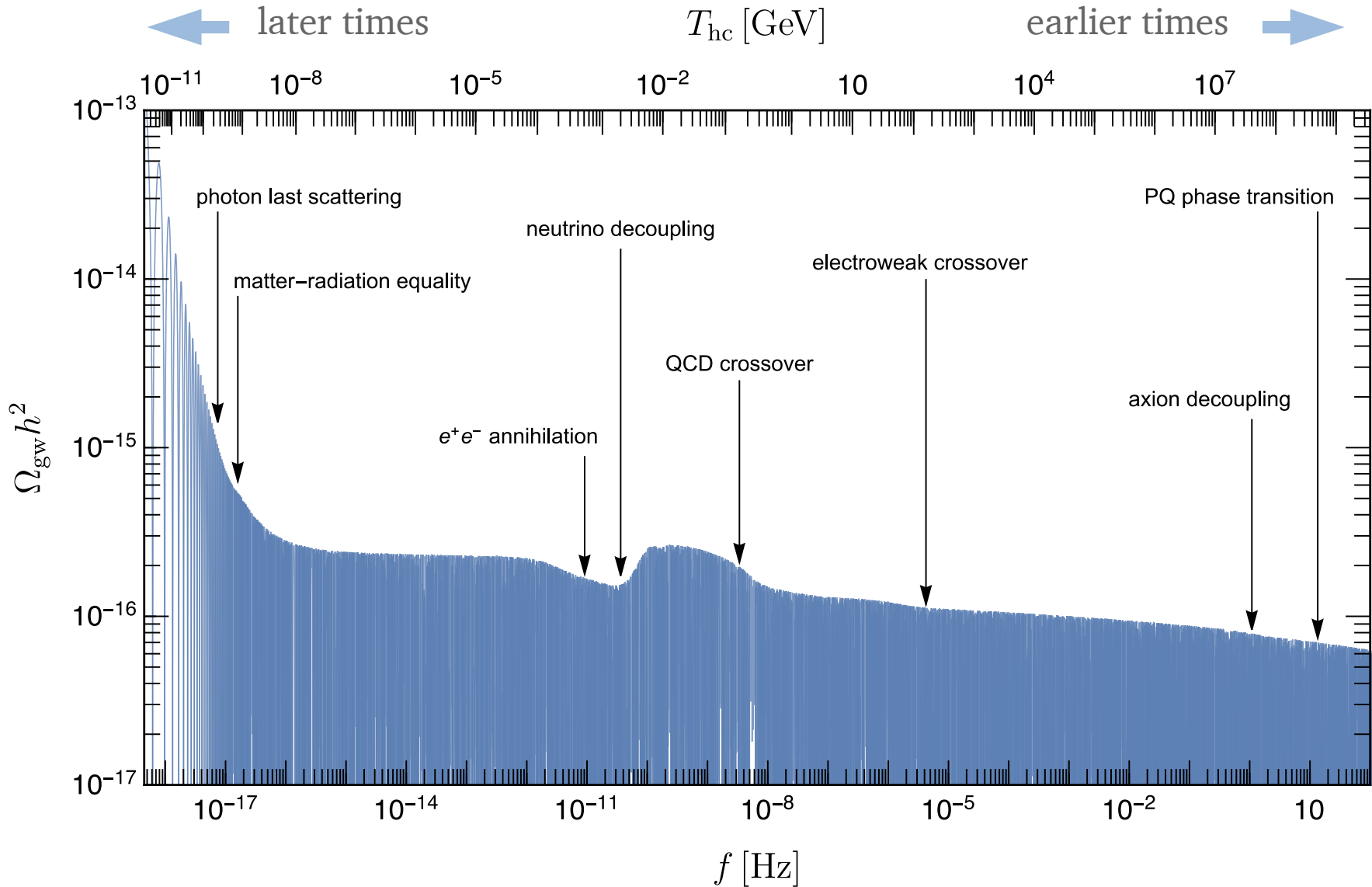
$$\frac{d^2 \chi(u)}{du^2} + \frac{2}{a(u)} \frac{da(u)}{du} \frac{d\chi(u)}{du} + \chi(u) = -24 \sum_{i=\gamma,\nu,a} F_i(u) \left[ \frac{1}{a(u)} \frac{da(u)}{du} \right]^2 \int_{u_i}^u dU \left[ \frac{j_2(u-U)}{(u-U)^2} \right] \frac{d\chi(U)}{dU},$$

$$\chi(0) = 1, \quad \chi'(0) = 1$$

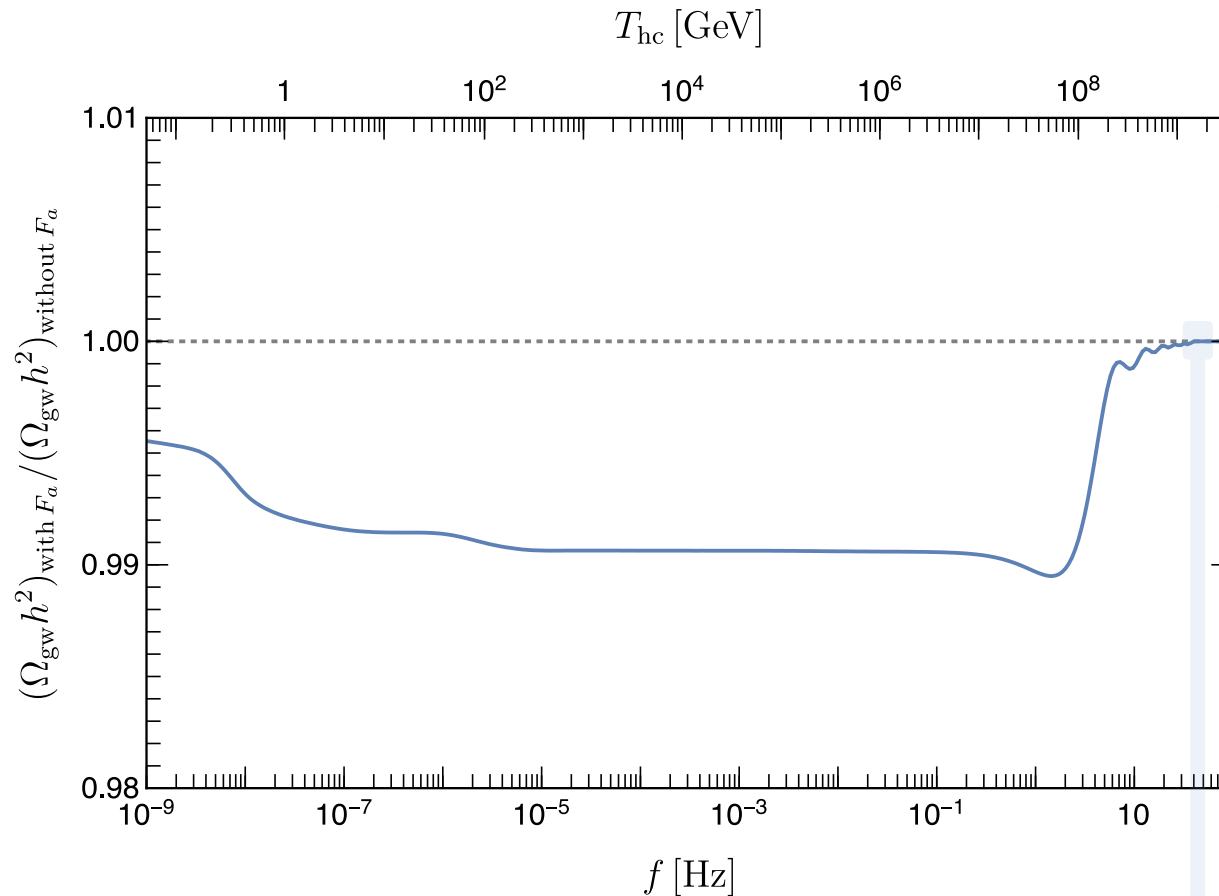
$u_i$ :  $u$  at Decoupling times:

$$T = \begin{cases} 3000 \text{ K} & (\gamma) \\ 2 \text{ MeV} & (\nu) \\ T_{\text{dec}} & (a) \end{cases}$$

# Birds-eye view of the frequency landscape



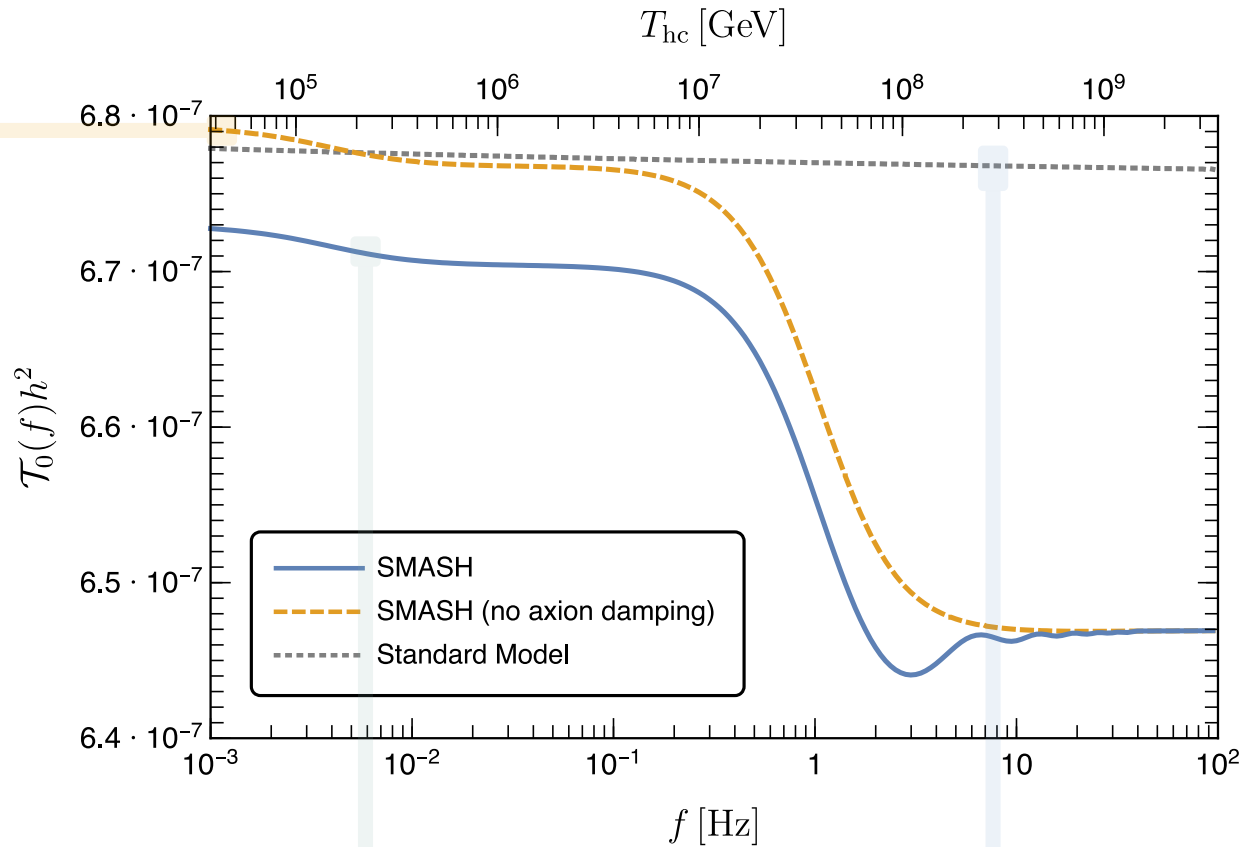
# Damping effect from free-streaming axions



Free streaming only after axion decoupling

**Suppression effect of 1%** below that of neutrinos (35% below  $10^{-10}$  Hz) and photons (14% below  $10^{-17}$  Hz)

# Where is the step?

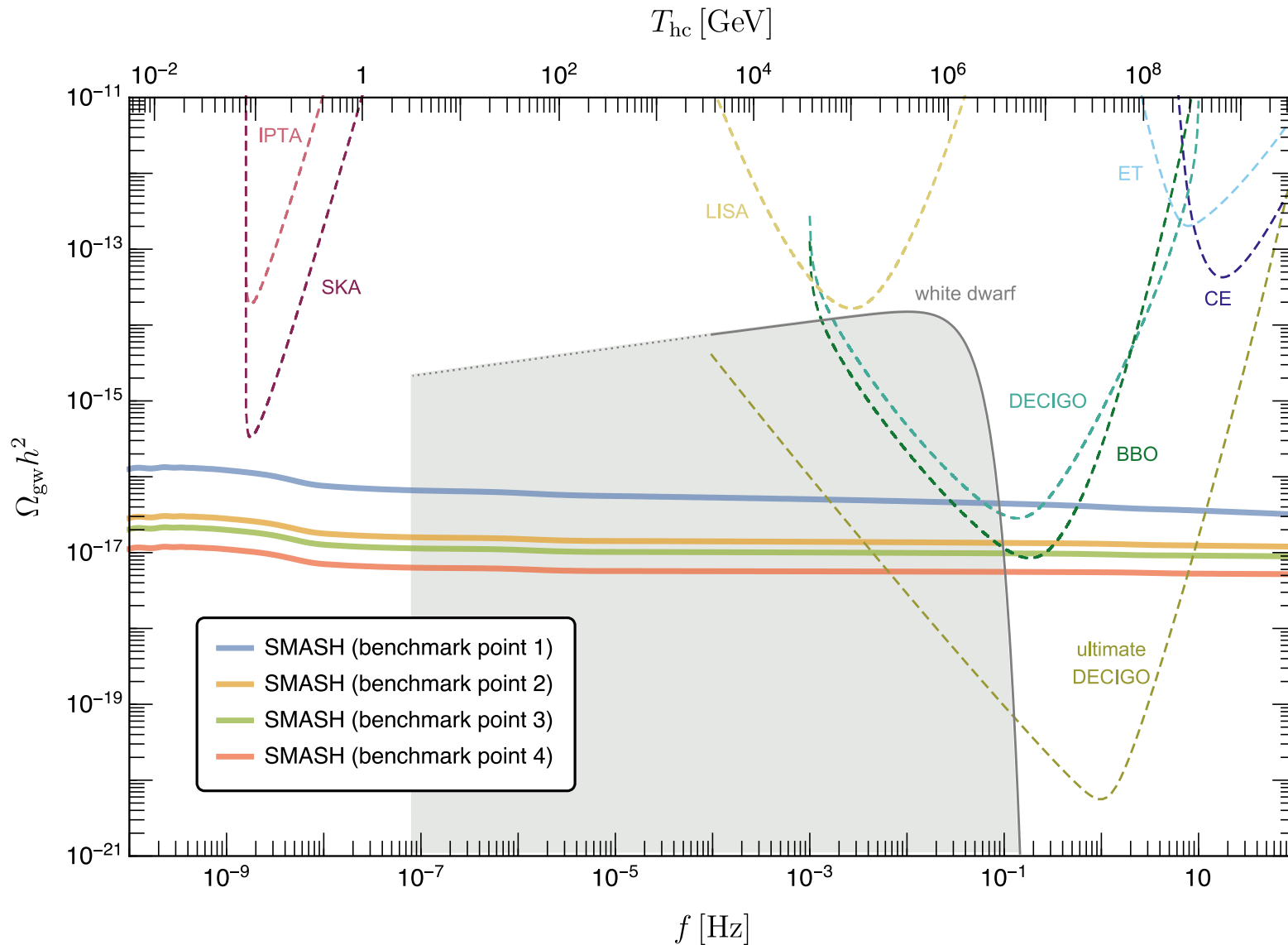


SM transfer function non-flat due to RG running

Smaller step due to decoupling of  $\text{Re}(\sigma)$

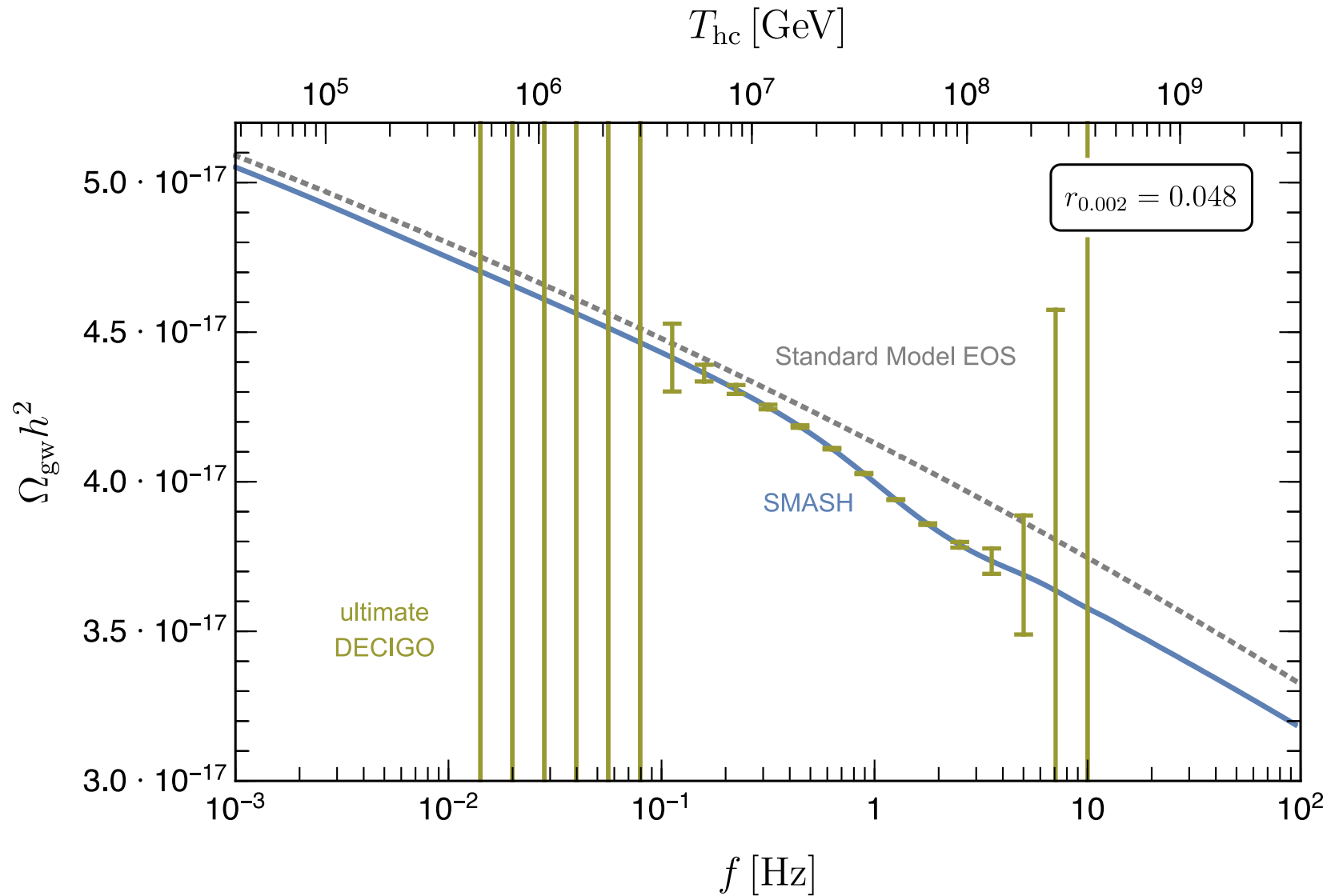
SMASH curve without damping above SM due to larger value of  $g_{*s}(T_0)$

# Future experimental sensitivities

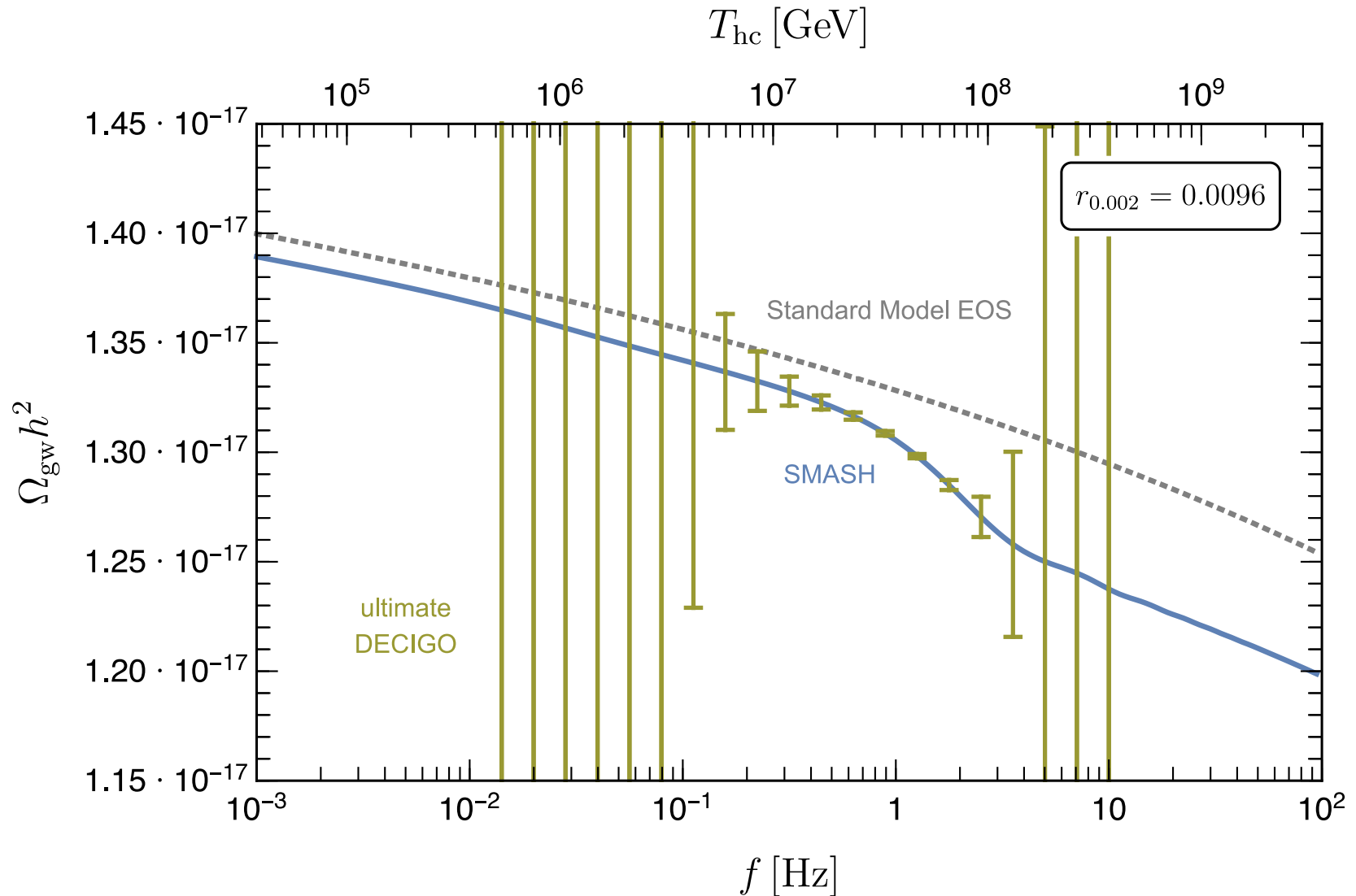




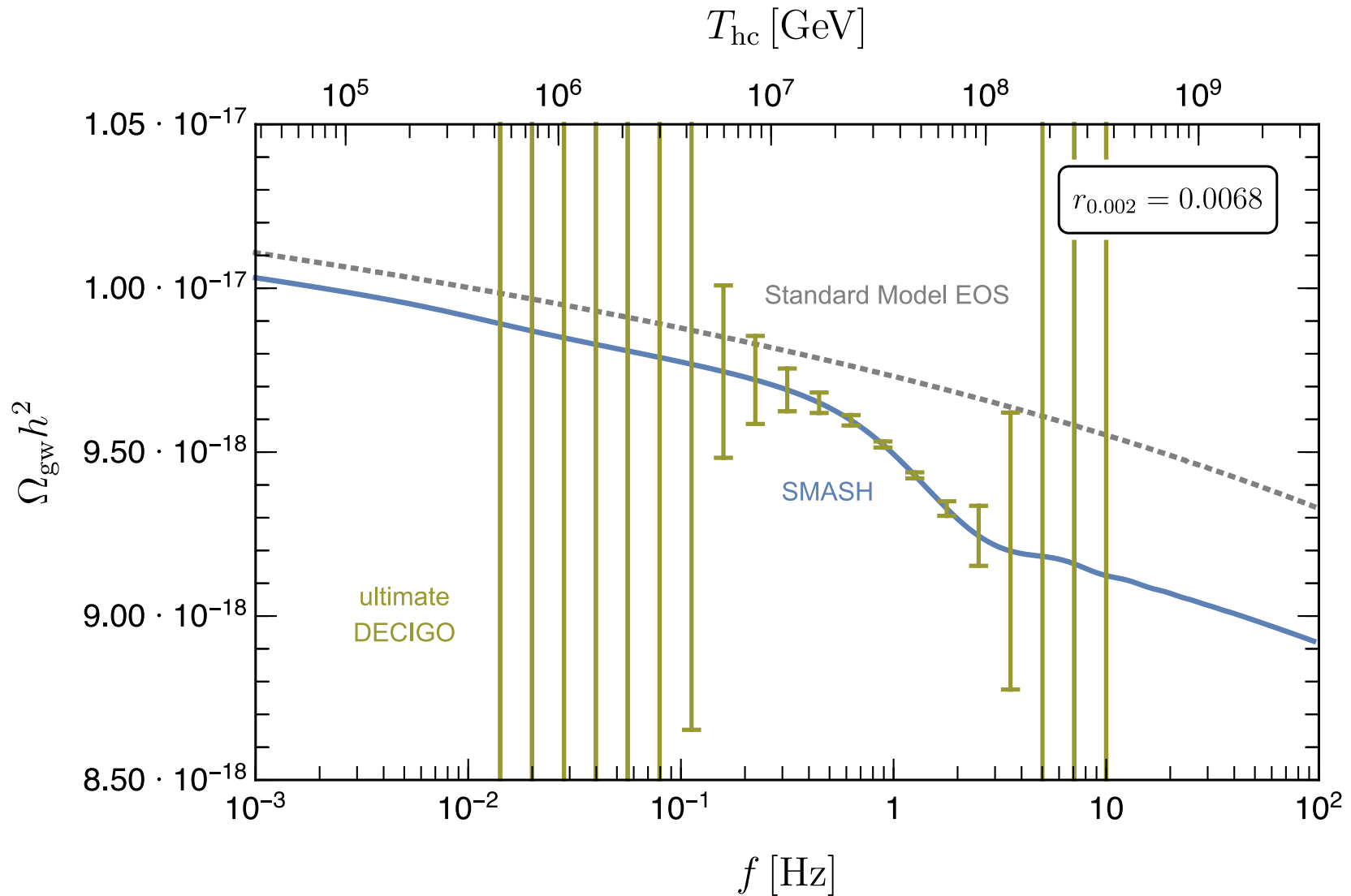
# Zooming into the signal region



# Zooming into the signal region



# Zooming into the signal region



# Conclusions

The **2<sup>nd</sup> order PQ phase transition** in **SMASH** predicts a **feature** in the **spectrum** of **primordial gravitational waves** near **1Hz**, corresponding to modes reentering the horizon when the temperature of the universe was  $T_{\text{hc}} \sim 10^8 \text{ GeV}$

This feature is just **above the white dwarf noise** and could be **observable** with the **ULTIMATE DECIGO** experiment as the feature is expected **near the peak sensitivity**

For our calculations of  $g_{*\rho}, g_{*s}$  across the phase transition we developed an **improved Daisy resummation** of thermal corrections which accounts for **decoupling** effects

Thank you!